Issn:0972-24

THE MATHEMATICS TEACHER

Volume-55

YEAR - 2019

ISSUSE-1&2

THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

The Association of Mathematics Teachers of India (AMTI) was started in 1965 for the promotion of efforts to improve Mathematics education at all levels. A major aim of the Association is to assist practising teachers of Mathematics in schools in improving their expertise and professional skills. Another important aim is to spot out and foster Mathematical talents in the young. The Association also seeks to disseminate new trends in Mathematics education among parents and public. Other activities of the Association include consultancy services to schools in equipping the Mathematics section of their libraries, in organising children's Mathematics clubs and fairs, in setting up teacher centres in schools, in conducting Mathematics laboratory programmes, in holding practical tests in Mathematics in assisting children in participating investigational projects.

The Association holds " National Mathematical Talent Search Competition " annually and organizes Orientation Courses, Seminars and Workshops for teachers and courses for talented students. A national conference is held annually in different parts of the country for teachers to meet and deliberate on important issues of Mathematics education. Innovative teacher award has been instituted to give public recognition to enterprising and pioneering teachers of Mathematics for which entries from teachers are invited.

An award for contributions to the Mathematics Teacher relating to History of Mathematics in the context of mathematics education had been instituted by Prof.R.C.Gupta.

"The Mathematics Teacher (India)" (MT) is the official quarterly journal of the Association and is issued twice a year. It has been approved for use in schools and colleges of education by the Government departments of education in many States. Besides MT the Association also brings out Junior Mathematician (JM), three issues in a year, especially for school students in English and Tamil.

The membership of the Association is open to all those interested in Mathematics and Mathematics Education. The membership fee inclusive of subscription for "**The Mathematics Teacher (India)**" and effective from April 1993 is as follows:

Subscription for India*

Category	Individual	Institutional
Life	Rs. 500	Rs. 1000
Annual (Ordinary)	Rs. 50	—
Junior Mathematician - Life	Rs. 250	Rs. 250
Junior Mathematician - Annual	Rs. 25	Rs. 25

The Journal "The Mathematics Teacher" will be supplied free to all members.
Fifty or more subscriptions to Junior Mathematician will entail 20% discount.

** For countries other than India same figures in US $.*
(inclusive of postage)
i.e instead of rupees read US dollars

The Mathematics Teacher
(INDIA)

OFFICIAL JOURNAL OF THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

Volume: 55 Issues: 1 & 2 Year: 2019

Dr. S.Muralidharan
Editor

EDITORIAL

This issue contains problems and solutions from the Mathematics competitions held during 2018.

Thanks to

for Partly supporting by grant (2016-17)
Science and Engineering Research Board (SERB),
(A statutory body of the Department of Science & Technology,
Government of India)

CONTENTS

1. Screening Test Gauss contest 3

2. Screening Test – Kaprekar contest 10

3. Screening Test – Bhaskara contest 20

4. Screening Test – Ramanujan contest 31

5. Final – Gauss contest 50

6. Final – Kaprekar contest 55

7. Final – Bhaskara contest 61

8. Final – Ramanujan contest 69

9. Pre RMO 80

10. RMO 88

11. RMO (TN and Kerala) 93

12. Indian National Mathematical Olympiad 98

13. Report on Association Activities 105

SCREENING TEST – GAUSS CONTEST
NMTC at PRIMARY LEVEL
V & VI Standards

PART – A

1. Observe the following sequence. What is the 100th term?

$$7, 8, 1, 0, 0, 1, 0, 1, 1, 0, 2, 1, 0, 3, \ldots$$

A. 1 B. 0 C. 2 D. 3

Solution Ignoring the first two numbers, the sequence is obtained from $101, 102, 103, 104, \ldots$ by separating the digits. Hence if we continue the sequence to 132, we would have written 98 terms $(3 \times 32 + 2)$ of the sequence and the 99th and 100th terms are from the number 133. Thus 100th term is 3 and answer is D.

2. A number is multiplied by 2 then by $\frac{1}{3}$, then by 4, then by $\frac{1}{5}$ then by 6 and finally by $\frac{1}{7}$. The answer is 16. Then the number is

A. odd B. even C. a square D. a cube

Solution When we multiply $2, \frac{1}{3}, 4, \frac{1}{5}, 6$ and $\frac{1}{7}$ we get $\frac{16}{35}$. We are given that when multiply a number by $\frac{16}{35}$ we obtain 16 and hence the number must be 35. Thus the number is odd and the answer is A.

3. Samrud bought a T-shirt for Rs 250. His friend Shlok wanted to buy it. Samrud wants to have a 10% profit on that. The selling price is (in rupees)

A. 280 B. 278 C. 276 D. 275

Solution Since 10% of 250 is 25, the selling price is $250 + 25 = 275$ Rs. Answer D.

4. The value of $1+21+41+61+81-11-31-51-71-91$ is

   A. -50 B. 50 C. 100 D. -100

   Solution We have

$$\begin{aligned}&1 + 21 + 41 + 61 + 81 - 11 - 31 - 51 - 71 - 91\\ &= (1 - 11) + (21 - 31) + (41 - 51) + (61 - 71) + (81 - 91)\\ &= -50\end{aligned}$$

   Answer A.

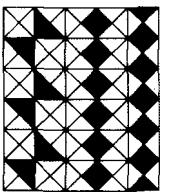


Figure 1

5. In the adjoining Figure 1 what portion of the figure is shaded?

   A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. $\frac{3}{4}$ D. $\frac{3}{10}$

   Solution The figure contains 30 squares and each can be divided into 4 triangles of equal areas by the diagonals of the square. Of the 120 triangles, 36 triangles are shaded. Hence the portion shaded is $\frac{36}{120} = \frac{3}{10}$. Answer D.

6. The sum of the numbers in the three brackets ( ) is

$$\frac{(\,)}{24} = \frac{20}{(\,)} = \frac{24}{18} = \frac{4}{(\,)}$$

A. 60 B. 55 C. 50 D. 45

Solution From $\frac{24}{18} = \frac{4}{()}$, we deduce that the number in the denominator of the last fraction is 3. Using this, we easily see that

$$\frac{(32)}{24} = \frac{20}{(15)} = \frac{24}{18} = \frac{4}{(3)}$$

Hence the sum of the numbers in the brackets is $32 + 15 + 3 = 50$. Answer C.

7. A is the smallest three digit number which leaves a remainder 2 when divided by 17. B is the smallest three digit number which leaves a remainder 7 when divided by 12. Then $A + B$ is

A. 205 B. 312 C. 215 D. 207

Solution 102 is the smallest three digit number that is divisible by 17. Hence the smallest three digit number that leaves a remainder 2 when divided by 17 is $102+2 = 104$. Similarly, the smallest three digit number that is divisible by 12 is 108 and hence 103 is the smallest three digit number that leaves a remainder 7 when divided by 12. Hence the required sum is $104 + 103 = 207$. Answer D.

8. A square of side 3 cm in cut into 9 equal squares. Another square of side 4 cm is cut into 16 equal squares. Saket made a bigger square using all the smaller square bits. The length of the side of the bigger square is (in cm)

A. 7 B. 6 C. 5 D. 8

Solution When we cut the square of side 3 cm into 9 equal squares, each of the resulting square has side 1 cm. Similarly, when we cut a square of side 4 cm into 16 equal squares, each square has side 1 cm. Thus we have 25 equal

squares with side 1 cm and we can form a square with side 5 cm with these. Answer C.

9. A contractor constructed a big hall, rectangular in shape, with length 32 meters and breadth 18 meters. He wanted to buy 1 meter by 1 meter tiles. But in the shop 3 meter by 2 meter tiles only were available. How many tiles he has to buy for tilting the floor?

   A. 48 B. 96 C. 120 D. 126

   Solution The area of the hall is 32×18 square meters. Each tile has area 3×2 square meters and hence the number of tiles required is $\frac{32 \times 18}{3 \times 2} = 96$. Answer B.

10. The fraction to be added to $2\frac{1}{3}$ to get the fraction $4\frac{4}{7}$ is

    A. $2\frac{1}{21}$ B. $2\frac{4}{21}$ C. $2\frac{5}{21}$ D. $2\frac{6}{21}$

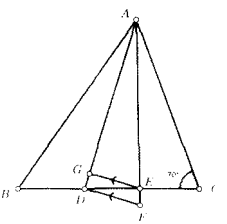
    Solution We have

$$4\frac{4}{7} - 2\frac{1}{3} = \frac{32}{7} - \frac{7}{3} = \frac{47}{21} = 2\frac{5}{21}$$

    we need to add $2\frac{5}{21}$ to $2\frac{1}{3}$ to get $4\frac{4}{7}$. Answer C.

PART - B

11. In the adjoining figure, $\angle BAD = \angle DAF = \angle FAC$. GE is parallel to DF, and $\angle EGA = 90°$. If $\angle ACE = 70°$ and $\angle BAC = 75°$, the measure of $\angle FDE$ is ———



    Solution $\angle EAC = \angle EAG = 25°$. Hence $\angle DFA = \angle EAC + \angle ACE = 95°$. Also, $\angle GEA = 90° - \angle GAE =$

$65°$. Thus $\angle GED = 95° - 65° = 30°$. Since $GE \parallel DF$, it follows that $\angle FDE = \angle GED = 30°$.

12. ABC is a triangle in which the angles are in the ratio $3 : 4 : 5$. PQR is a triangle in which the angles are in the ratio $5 : 6 : 7$. The difference between the least angle of ABC and the least angle of PQR is $a°$. Then $a =$ ———

Solution The angles of the triangle ABC are $45°, 60°, 75°$ and those of the triangle PQR are $50°, 60°, 70°$. Hence the difference between the least angle of ABC and the least angle of PQR is $50° - 45° = 5°$ and $a = 5$.

13. Samrud had to multiply a number by 35. By mistake he multiplied by 53 and got a result 720 more. The new product is ———

Solution The difference between 53 and 35 is 18 and hence 18 times the number that was to be multiplied is 720. Thus the number is 40 and the new product is $40 \times 53 = 2120$.

14. Vishva plays football every 4th day. He played on a Tuesday. He plays football on a Tuesday again in ——— days.

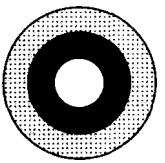
Solution A Tuesday occurs once in 7 days and Vishva plays football every 4th day. If he played on a Tuesday, the next Tuesday he will play will be in 28 days (the least common multiple of 7 and 4).

15. In an elementary school 26% of the students are girls. If there are 240 less girls than boys, then the strength of the school is ———

Solution The percentage of boys is 74% and hence

the difference in the percentage of boys and girls is $74\% - 26\% = 48\%$. Thus 48% of school strength is 240 and hence the strength of the school is $\dfrac{100 \times 240}{48} = 500$.

16. There are three concentric circles as shown in the figure. The radii of them are 2 cm, 4 cm and 6 cm. The ratio of the area of the shaded region to the area of the dotted region is $\frac{a}{b}$ where a, b are integers and have no common factor other than 1. Then $a + b =$ ———



Solution The area of the shaded region equals the difference in areas of circles of radii 4 cm and 2 cm and hence is $16\pi - 4\pi = 12\pi$. The area of the dotted portion equals the difference in areas of circles of radii 6 cm and 4 cm and hence is $36\pi - 16\pi = 20\pi$. Thus the ratio of the areas is $\dfrac{a}{b} = \dfrac{12\pi}{20\pi} = \dfrac{3}{5}$. Thus $a + b = 8$.

17. The value of

$$\left(1+\frac{1}{9}\right)\left(1+\frac{1}{8}\right)\left(1+\frac{1}{7}\right)\left(1+\frac{1}{6}\right) \times \left(1+\frac{1}{5}\right)\left(1+\frac{1}{4}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{2}\right)$$

is ———

Solution The given product is

$$\frac{10}{9}\cdot\frac{9}{8}\cdot\frac{8}{7}\cdot\frac{7}{6}\cdot\frac{6}{5}\cdot\frac{5}{4}\cdot\frac{4}{3}\cdot\frac{3}{2} = 5$$

18. When a two digit number divides 265, the remainder is 5. The number of such two digit numbers is ———

Solution Since the remainder is 5, the two digit number must divide $265 - 5 = 260$. Now, $260 = 2 \times 2 \times 5 \times 13$.

The possible two digit numbers that divide 260 are, therefore, $10, 13, 20, 26, 52, 65$ and hence there are six such numbers.

19. If $A\#B = \dfrac{A \times B}{A + B}$, the value of $\dfrac{12\#8}{8\#4} + \dfrac{10\#6}{6\#2}$ is ———

Solution We have

$$12\#8 = \frac{12 \times 8}{12 + 8} = \frac{96}{20} = \frac{24}{5}$$

$$8\#4 = \frac{8 \times 4}{8 + 4} = \frac{32}{12} = \frac{8}{3}$$

$$10\#6 = \frac{10 \times 6}{10 + 6} = \frac{60}{16} = \frac{15}{4}$$

$$6\#2 = \frac{6 \times 2}{6 + 2} = \frac{12}{8} = \frac{3}{2}$$

Hence

$$\frac{12\#8}{8\#4} + \frac{10\#6}{6\#2} = \frac{24/5}{8/3} + \frac{15/4}{3/2} = \frac{9}{5} + \frac{5}{2} = \frac{43}{10}$$

20. When water becomes ice, its volume increases by 10%. When ice melts into water its volume decreases by $a\,\%$. Then $a =$ ———

Solution When water becomes ice, volume increases by 10% - that is the volume of ice is $1.1\times$ volume of water. Hence when ice melts, the volume becomes $\dfrac{1}{1.1}$ times the volume of ice. Thus $a = 100 - \dfrac{100}{1.1} = 9\,\dfrac{9}{11}\%$.

SCREENING TEST – KAPREKAR CONTEST
NMTC at SUB JUNIOR LEVEL
VII & VIII STANDARDS

PART - A

1. The fraction greater than $8\frac{4}{9}$ is

   A. $8\frac{1}{3}$ B. $\frac{150}{18}$ C. $8\frac{2}{3}$ D. $\frac{216}{27}$

   Solution Clearly, $8\frac{1}{3} = 8\frac{3}{9} < 8\frac{4}{9}$. Also, $\frac{150}{18} = \frac{25}{3} = 8\frac{1}{3}$ and hence this is also less than $8\frac{4}{9}$. Again, $\frac{216}{27} = 8$. Since $\frac{2}{3} = \frac{6}{9} > \frac{4}{9}$, it follows that only $8\frac{2}{3} > 8\frac{4}{9}$. Answer C.

2. A car is slowly driven in a road full of fog. The car passes a man who was walking at the rate of 3 km an hour in the same direction. He could see the car for 4 minutes and was visible for up to a distance of 100 meters. The speed of the car is (in km per hour)

   A. $4\frac{1}{2}$ B. 4 C. $3\frac{1}{2}$ D. 3

   Solution The car moves 100 meters in 4 minutes and hence 1500 meters in 60 minutes. Thus the relative speed of the car is 1.5 km per hour. Since the man's walking speed is 3 km per hour, the speed of the car is 3+1.5 = 4.5 km per hour. Answer A.

3. Kiran sells pens at a profit of 20% for Rs 60. But due to lack of demand he reduced its price to Rs 55. Then

   A. he gets a profit of 10% B. he gets a profit of 12%

   C. he incurs a loss of 10% D. he incurs a loss of 8%

   Solution If C is the cost price of the pen, we are given that $1.2C = 60$. Thus $C = 50$. If the pen is sold at Rs

55, the profit is 5 Rs and hence the profit percentage is $\frac{5}{50} \times 100 = 10\%$. Answer A.

4. If 40% of a number is added to another number then it becomes 125% of itself. The ratio of the second to the first number is

   A. 5 : 8 B. 7 : 5 C. 8 : 5 D. None of these

   Solution Let the numbers be a, b. Given that $0.4a+b = 1.25a$. Thus $0.85a = b$ and the ratio $a : b = 20 : 17$. Answer D.

5. The length of a rectangular sheet of paper is 33 cm. It is rolled along its length into a cylinder so that its width becomes height of the cylinder. The volume is 1386 cubic cms. The width of the rectangular sheet (in cm) is

   A. 14 B. 15 C. 16 D. 18

   Solution Since the length becomes the circular base with radius r of the cylinder, we have $2\pi r = 33$ and hence $r = \frac{33}{2\pi} = \frac{21}{4}$. If w is the width of the rectangle, we have

$$1386 = \pi \left(\frac{21}{4}\right)^2 w \Rightarrow w = 1386 \times \frac{16}{441} \times \frac{7}{22} = 16$$

   Answer C.

6. If

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \cdots + \frac{1}{n \times (n+1)} = \frac{19}{20}$$

   then $n =$

   A. 18 B. 19 C. 20 D. 25

   Solution Observe that $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$. Hence

the give sum can be written as

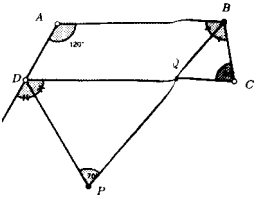
$$\left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\cdots+\left(\frac{1}{n+1}-\frac{1}{n}\right)$$
$$=1-\frac{1}{n+1}$$
$$=\frac{n}{n+1}=\frac{19}{20}$$

Thus $n=19$. Answer B.

7. a, b are natural numbers. If $9a^2 = 12a + 96$ and $b^2 = 2b + 3$, the value of $2018(a+b)$ is
A. 14226 B. 14128 C. 14126 D. 14246
Solution a satisfies $3a^2 - 4a - 32 = 0$. This can be factored as $(a-4)(3a+8) = 0$. Since a is a natural number, it follows that $a = 4$. Now, b satisfies $b^2-2b-3=0$. This can be factored as $(b-3)(b+1)=0$. Hence $b=3$. Consequently, $2018(a+b) = 2018(3+4) = 14126$. Answer C.

8. Shanti has three daughters. The average age of them is 15 years. Their ages are in the ratio $3:5:7$. The age of the youngest daughter is (in years)
A. 8 B. 9 C. 10 D. 12
Solution Let their ages be $3x, 5x, 7x$. The average age is $\frac{15x}{3} = 15$ and hence $x = 3$. The age of the youngest is $3x = 9$ years. Answer B.

9. In the adjoining figure, $ABCD$ is a quadrilateral. The bisectors of $\angle B$ and the exterior angle at D meet at P. Given $\angle C = 80°, \angle ADC = \frac{1}{2}\angle A$ and $\angle A = \angle C + 40°$. Then $\angle DPB$ is



A. 50° B. 60° C. 70° D. 80°

Solution Let DP and AC meet at Q. Since $\angle C = 80^\circ$ and $\angle A = \angle C + 40^\circ$, we have $\angle A = 120^\circ$. Also, $\angle ADC = \frac{1}{2}\angle A = 60^\circ$. The exterior angle at D equals 120° and thus $\angle CDP = 60^\circ$. Now, $\angle B = 360^\circ - 80^\circ - 120 - 60^\circ = 100^\circ$ and it follows that $\angle DQP = \angle BQC = 180^\circ - 80^\circ - 50^\circ = 50^\circ$. Now, $\angle DPB = 180^\circ - 50^\circ - 60^\circ = 70^\circ$. Answer C.

10. The number of 3-digit numbers which contain 6 and 7 is
A. 52 B. 60 C. 62 D. 64
Solution First, we count the number of three digit numbers whose leading digit is neither 6 or 7. The first digit can be filled in 7 ways (any digit other than $0, 6, 7$) and the last two digits are either 67 or 76. Thus there are 14 such numbers.

We now count the numbers with leading digit 6. The digit 7 must be one of the other two digits and the other digit can be any digit. Thus there are 20 such numbers. But in this we have counted 677 twice. Thus there 19 numbers that start with 6. Similarly, there are 19 numbers with leading digit 7. Hence we have a total of $14+19+19 = 52$ numbers.

We can also see this by *counting the complement* – counting the numbers that do not contain 6 or 7 and subtracting this count from the total number of three digit numbers.

The three digit numbers that do not contain 6 are $8 \times 9 \times 9 = 648$. Similarly the numbers that do not contain 7 are 648. In this we have counted the numbers that neither contain 6 nor 7 twice. The count of such numbers is $7 \times 8 \times 8 = 448$. Thus the count of numbers that contain 6 or 7 is $648 + 648 - 448 = 848$. Since

the total number of three digit numbers is 900, there are $900 - 848 = 52$ numbers that contain both 6 and 7. Answer A.

11. The difference between the biggest and the smallest three digit numbers each of which has different digits is
A. 864 B. 875 C. 885 D. 895
Solution The largest three digit number with different digits is 987 and the smallest number with different digits is 102. The difference is therefor $987 - 102 = 885$. Answer C.

12. If $3x + 1 = 2y - 1 = 5z + 3 = 7w + 1 = 15$, the value of $6x - 3y + 5z - 8w$ is
A. 1 B. 2 C. 3 D. None of these
Solution From the given equations, we see that $x = \frac{14}{3}, y = 8, z = \frac{12}{5}$ and $w = 2$. Thus $6x - 3y + 5z - 8w = 28 - 24 + 12 - 16 = 0$. Answer D.

13. Five years ago the average age of Aruna, Roy, David and Salman is 45 years. Sita joins them now. The average age of all the five now is 49 years. The present age of Sita is (in years)
A. 45 B. 43 C. 51 D. 48
Solution If the current ages of Aruna, Roy, David and Salman are respectively a, b, c and d, then given that

$$\frac{(a-5)+(b-5)+(c-5)+(d-5)}{4} = 45$$

If e is the present age of Sita, then

$$\frac{a+b+c+d+e}{5} = 49$$

Hence $a+b+c+d = 4\times45+20 = 200$ and $a+b+c+d+e = 5 \times 49 = 245$. Hence $e = 45$ and Sita's present age is 45 years. Answer A.

14. The fraction $\dfrac{B}{3x-1}$ is subtracted from the fraction $\dfrac{A}{2x+3}$. The resulting fraction is $\dfrac{-11}{(2x+3)(3x-1)}$. Then $A+B=$

A. 11 B. -11 C. 5 D. -5

Solution Given

$$\frac{A}{2x+3}-\frac{B}{3x-1}=\frac{-11}{(2x+3)(3x-1)}$$

Noting that $2x+3=1-3x$ when $x=-\dfrac{2}{5}$, substituting $x=-\dfrac{2}{5}$, we get

$$\frac{A}{11/5}+\frac{B}{11/5}=\frac{-11}{(11/5)\cdot(-11/5)}$$
$$\frac{5(A+B)}{11}=\frac{25}{11}$$

and hence $A+B=5$. Answer C.

15. There are some cows and ducks. The total number of legs is equal to 14 more than twice the number of heads. The number of cows is

A. 5 B. 6 C. 7 D. 8

Solution Let C be the number of cows and D, the number of ducks. Number of legs is $4C+2D$ and the number of heads is $C+D$. Thus we have $4C+2D=14+2(C+D)$. Hence $C=7$. Answer C.

16. The sum of 5% of a number and 9% of another number is equal to sum of the 8% first number and 7% of the second number. The ratio between the numbers is

A. 3 : 2 B. 5 : 7 C. 7 : 9 D. 2 : 3

Solution Let the numbers be a, b. We have

$$0.05a+0.09b=0.08a+0.07b$$

and hence $0.03a=0.02b$. Thus $a:b=2:3$. Answer D.

17. The length of two sides of an isosceles triangle are 8 cm and 14 cm. The perimeter of the triangle (in cm) is
A. 30 B. 36 C. 19 D. 30 or 36
Solution The equal sides of the isosceles triangle can have lengths 8 or 14. In the first case, the perimeter is $8+8+14=30$ and in the second case, the perimeter is $8+14+14=36$. Answer D.

18. There are three cell phones A, B, C. A is 50% costlier than C and B is 25% costlier than C. A is $a\%$ costlier than B. Then $a=$
A. 25 B. 20 C. 15 D. 10
Solution Let their costs be x, y, z respectively. We have $x=1.5z, y=1.25z$. Thus $x=1.5z=1.5\times\dfrac{y}{1.25}=\dfrac{6}{5}y=1.2y$. Hence A is 20% costlier than B. Answer B.

19. Sushant wrote a two digit number. He added 5 to the tens digit and subtracted 3 from the unit digit of the number and got a number equal to twice the original number. The original number is
A. 47 B. 74 C. 37 D. 73
Solution Let the two digit number be $10a+b$. Given

$$\begin{aligned} & 10(a+5)+(b-3)=2(10a+b) \\ \Rightarrow\ & 10a+b+47=20a+2b \\ \Rightarrow\ & 10a+b=47 \end{aligned}$$

Answer A.

20. The units digit of $5^{2018}-3^{2018}$ is
A. 5 B. 6 C. 7 D. 4
Solution The number 5^n for any positive integer $n\geq 2$ ends with 25. The units digits of 3^n, for $n=1,2.3,\ldots$ are $3,9,7,1,3,9,7,1,\ldots$. Thus the units digit of 3^{2018} is 9 since 2018 is 2 more than a multiple of 4. Since

$25-9=16$, it follows that the units digit of $5^{2018}-3^{2018}$ is 6. Answer B.

PART – B

21. The smallest natural number that has to be added to 803642 to get a number which is divisible by 9 is ———
Solution The remainder when 803642 is divided by 9 is 5 and hence the smallest number that needs to be added to get a multiple of 9 is 4.

22. The greatest two digit number that will divides 398, 436 and 542 leaving respectively 7, 11 and 15 as remainders is ———
Solution Such number will be a divisor of $398-7=391, 436-11=425$ and $542-15=527$. Hence it must also be a divisor of $425-391=34$ and $527-425=102$. Hence the desired number must be a divisor of 34. The remainder when 34 divides 398 is 21 and hence does not satisfy the conditions. On the other hand, the remainder when 17 divides 398 is 7, when it divides 436, it is 11 and when it divides 542, the remainder is 15. Thus 17 is the desired two digit number.

23. $\frac{2}{3}$ is ———% *of* $\frac{1}{3}$.
Solution We need to find x such that $\frac{x}{100}\times\frac{1}{3}=\frac{2}{3}$. Hence $x=200$.

24. The sum of 5 positive integers is 280. The average of the first 2 numbers is 40. The average of the third and fourth number is 60. The fifth number is ———
Solution The sum of the first two numbers is 80 and the sum of the third and fourth numbers is 120. Hence the sum of the first four numbers is 200 and consequently, the fifth number equals $280-200=80$.

25. If $a : b = 3 : 4$ and $\frac{p}{q} = \frac{a^2 + b^2 + ab}{a^2 + b^2 - ab}$, where p, q have no common divisors other than 1, $p + q$ is ———
Solution Putting $a - 3k$ and $b = 4k$ m we have

$$\frac{a^2 + b^2 + ab}{a^2 + b^2 - ab} = \frac{9k^2 + 16k^2 + 12k^2}{9k^2 + 16k^2 - 12k^2} = \frac{37}{13}$$

Thus $p + q = 50$.

26. a is a natural number such that a has exactly two divisors and $(a + 1)$ has exactly three divisors. The number of divisors of $a + 2$ is ———
Solution A number has exactly two divisors if and only if it is a prime. Also, a number has exactly three divisors if and only if it is the square of a prime number. Thus $a + 1 = p^2$ and hence $a = (p - 1)(p + 1)$. Since a is a prime, it follows that $p - 1 = 1$ and $a = 3$. Thus $a + 2 = 5$ and it has exactly 2 divisors.

27. The first term of a series is $\frac{2}{5}$. If x is a term of this series, the next term is $\frac{1 - x}{1 + x}$. If t_n denotes the n-th term and $t_{2018} - t_{2017} = \frac{p}{q}$, where p, q are integers having no common factors other than 1, $p + q$ is ———
Solution When $t_k = x$, we have

$$\begin{aligned} t_{k+1} &= \frac{1 - x}{1 + x} \\ t_{k+2} &= \frac{1 - t_{k+1}}{1 + t_{k+1}} \\ &= \frac{1 - \dfrac{1 - x}{1 + x}}{1 + \dfrac{1 - x}{1 + x}} \\ &= x \end{aligned}$$

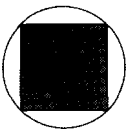
Hence the sequence is $\frac{2}{5}, \frac{3}{7}, \frac{2}{5}, \ldots$. Thus $t_{2018} = \frac{3}{7}$ and

$t_{2017} = \frac{2}{5}$. Hence

$$t_{2018} - t_{2017} = \frac{3}{7} - \frac{2}{5} = \frac{1}{35}$$

and $p + q = 36$.

28. In the adjoining figure, the side of the square is $\sqrt{\frac{2018}{\pi}}$ cm. The area of the unshaded region is $\left(\frac{\pi - 2}{\pi}\right) A$ sq cms. The value of A is ———



Solution If d is the side of the square, the length of the diagonal is $d\sqrt{2}$. The radius of the circle is $\frac{d\sqrt{2}}{2} = \frac{d}{\sqrt{2}}$. Thus the area of the unshaded portion is

$$\pi\frac{d^2}{2} - d^2 = d^2\left(\frac{\pi - 2}{2}\right) = \frac{2018}{\pi}\left(\frac{\pi - 2}{2}\right)$$

Thus $A = \frac{2018}{2} = 1009$.

29. n is a natural number. The square root of the sum of the square of n and 19 is equal to the next natural number to n. The value of n is ———

Solution Given $\sqrt{n^2 + 19} = n + 1$. Thus $2n + 1 = 19$ and $n = 9$.

30. Using only the digits 1, 2, 4, 5, two-digit numbers are formed. The digits of the two digit number may be the same or different. The number of such two-digit number is ———

Solution The tenths digit can be any of the given four numbers and the units digits can again be any of the four numbers (since digits may be same). Thus the number of such two digit numbers is 16.

SCREENING TEST – BHASKARA CONTEST
NMTC at JUNIOR LEVEL
IX & X Standards

PART - A

1. The value of $\dfrac{3+\sqrt{6}}{8\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}-\sqrt{27}}$ is

   A. $\sqrt{2}$ B. $\sqrt{3}$ C. $\sqrt{6}$ D. $\sqrt{18}$

   Solution

$$\begin{aligned}
&\frac{3+\sqrt{6}}{8\sqrt{3}-2\sqrt{12}-\sqrt{32}+\sqrt{50}-\sqrt{27}} \\
&\quad=\frac{3+\sqrt{6}}{8\sqrt{3}-4\sqrt{3}-4\sqrt{2}+5\sqrt{2}-3\sqrt{3}} \\
&\quad=\frac{3+\sqrt{6}}{\sqrt{3}+\sqrt{2}} \\
&\quad=\frac{\sqrt{3}(\sqrt{3}+\sqrt{2})}{\sqrt{3}+\sqrt{2}} \\
&\quad=\sqrt{3}
\end{aligned}$$

   Answer B.

2. A train moving with a constant speed crosses a stationary pole in 4 seconds and a platform 75 m long in 9 seconds. The length of the train is (in meters)

   A. 56 B. 58 C. 60 D. 62

   Solution If L is the length of the train and s its speed in meters per second, then we have $L = 4s$ and $(75 + L) = 9s$. This gives $s = 15$ and $L = 60$ meters. Answer C.

3. One of the factors of $9x^2 - 4z^2 - 24xy + 16y^2 + 20y - 15x + 10z$ is

   A. $3x - 4y - 2z$ B. $3x + 4y - 2z$ C. $3x + 4y + 2z$

D. $3x - 4y + 2z$

Solution

$$\begin{aligned} &9x^2 - 4z^2 - 24xy + 16y^2 + 20y - 15x + 10z \\ &= (9x^2 - 24xy + 16y^2) - 4z^2 - 15x + 20y + 10z \\ &= (3x - 4y)^2 - 4z^2 - 15x + 20y + 10z \\ &= (3x - 4y - 2z)(3x - 4y + 2z - 5) \end{aligned}$$

Answer A.

4. The natural number which is subtracted from each of the four numbers 17, 31, 25, 47 to give four numbers in proportion is

A. 1 B. 2 C. 3 D. 4

Solution Let the number that is subtracted be x. We have $17-x, 31-x, 25-x, 47-x$ are in proportion. Hence $\dfrac{17-x}{31-x} = \dfrac{25-x}{47-x}$. Solving for x, we get $x = 3$.

Answer C.

5. The solution to the equation $5\,(3^x) + 3\,(5^x) = 510$ is

A. 2 B. 4 C. 5 D. No solution

Solution We have $3(5^x) = 5(102 - 3^x)$. Noting that $102 - 3^x$ is not a multiple of 3 for any positive integer x, there is no solution in positive integers. If x is negative, then we have $3^{-x+1} + 5^{-x+1} = 510 \cdot 15^{-x}$. The right hand side is a multiple of 3 where as the left hand side is not a multiple of 3. Thus x can't be negative integer either. Hence the equation has no solution.

Alternately, we can also verify that $x = 2, 4$ or 5 does not satisfy the given equation. Answer D.

6. If $(x + 1)^2 = x$, the value of $11x^3 + 8x^2 + 8x - 2$ is

A. 1 B. 2 C. 3 D. 4

Solution From $(x + 1)^2 = x$, we have $x^2 + 2x + 1 = x$

and hence $x^2 + x = -1$.

$$\begin{aligned} 11x^3 + 8x^2 + 8x - 2 &= 11x^3 + 8(x^2 + x) - 2 \\ &= 11x^3 - 10 \\ &= 11x^2(x+1)^2 - 10 \\ &= 11(x^2 + x)^2 - 10 \\ &= 11 - 10 = 1 \end{aligned}$$

Answer A.

7. There are two values of m for which the equation $4x^2 + mx + 8x + 9 = 0$ has only one solution for x. The sum of these two values of m is
A. -16 B. 16 C. 8 D. 0
Solution The discriminant of the given quadratic is $(m+8)^2 - 144$ and this needs to be zero, if there is only one solution for x. Hence $m^2 + 16m - 80 = 0$ and thus the sum of the values of m is -16. Answer A.

8. The number of zeros in the product of the first 100 natural numbers is
A. 12 B. 15 C. 18 D. 24
Solution Let $N = 1 \cdot 2 \cdot 3 \cdots 100$. The highest power of 5 that divides N is given by

$$\left\lfloor \frac{100}{5} \right\rfloor + \left\lfloor \frac{100}{5^2} \right\rfloor = 20 + 4 = 24$$

Similarly the highest power of 2 that divides N is

$$\left\lfloor \frac{100}{2} \right\rfloor + \left\lfloor \frac{100}{2^2} \right\rfloor + \cdots = 50 + 25 + 12 + 1 = 88$$

Hence the highest power of 10 that divides N is 24 and hence it ends with 24 zeroes. Answer D.

9. The length of each side of a triangle is increased by 20% then the percentage increase of area is

A. 60% B. 120% C. 80% D. 44%

Solution The area of triangle with sides a, b, c is given by

$$\sqrt{s(s-a)(s-b)(s-c)}$$

where $a = \dfrac{a+b+c}{2}$. When each of a, b, c are increased by 20%, s also increases by 20% and the new area is

$$\sqrt{1.2s(1.2s-1.2a)(1.2s-1.2b)(1.2s-1.2c)}$$
$$= 1.44\sqrt{s(s-a)(s-b)(s-c)}$$

Hence the area increases by 44 %. Answer D.

10. The number of pairs of relatively prime positive integers (a, b) such that $\dfrac{a}{b} + \dfrac{15b}{4a}$ is an integer is

A. 1 B. 2 C. 3 D. 4

Solution If $\dfrac{a}{b} + \dfrac{15b}{4a} = k$, an integer, we have $15b^2 = 4a(kb - a)$ and hence a must divide 15. Also, 4 must divide b^2. The possible values of a are $1, 3, 5, 15$ and b can be any even number. The only possible value that satisfies the given condition is $b = 2$. Similarly, we see that when $a = 3, 5$ or 15, the possible value for b is 2. Thus we have four solutions $(1, 2), (3, 2), (5, 2)$ and $(15, 2)$.

11. The four digit number $8ab9$ is a perfect square. The value of $a^2 + b^2$ is

A. 52 B. 62 C. 54 D. 68

Solution Since the given number ends with 9, its square root ends with either 3 or 7. Again, the leading digit is 8. Thus the tenths digit of the square root is either 8 or 9. We have $87^2 = 7569$ and $93^2 = 8649$. Thus $ab = 64$ and $a^2 + b^2 = 36 + 16 = 52$. Answer A.

12. a, b are positive real numbers such that $\dfrac{1}{a} + \dfrac{9}{b} = 1$. The smallest value of $a + b$ is

A. 15 B. 16 C. 17 D. 18

Solution Note that $a > 1$. We have $\frac{9}{b} = 1 - \frac{1}{a} = \frac{a-1}{a}$. Hence $b = \frac{9a}{a-1} = 9 + \frac{9}{a-1}$. Thus

$$a + b = a + 9 + \frac{9}{a-1} = 10 + (a-1) + \frac{9}{a-1}$$

By AM - GM inequality, $(a-1) + \frac{9}{a-1} \geq 6$ and hence the least value of $a + b$ is 16. Answer B.

13. a, b are real numbers. The least value of $a^2 + ab + b^2 - a - 2b$ is

A. 1 B. 0 C. -1 D. 2

Solution Let k be the least value of the given expression. Then

$$a^2 + ab + b^2 - a - 2b - k \geq 0$$
$$\Rightarrow a^2 + a(b-1) + (b^2 - 2b - k) \geq 0$$

Hence, as a quadratic in a, the discriminant must be non positive. Hence

$$(b-1)^2 - 4(b^2 - 2b - k) \leq 0$$
$$\Rightarrow -3(b-1)^2 + 4(k+1) \leq 0$$

The least value of k for which the above inequality holds for all real values of b is $k = -1$. Note that when $a = 0$ and $b = 1$, the expression equals -1. Thus the least value is -1. Answer C.

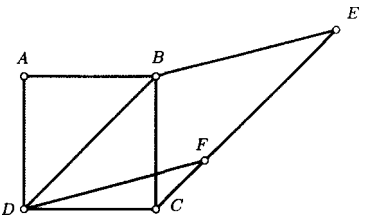
14. I is the incenter of a triangle ABC in which $\angle A = 80°$. $\angle BIC =$

A. 120° B. 110° C. 125° D. 130°

Solution Since I is the incenter, $\angle BIC = 90° + \frac{1}{2}\angle A = 130°$. Answer D.

15. In the adjoining figure, $ABCD$ is a square and $DFEB$ is a rhombus. $\angle CDF =$

A. $15°$ B. $18°$ C. $20°$ D. $30°$



Solution In triangle BCF, $\angle BCF = 135°$. If $CF = x$, by Cosine formula, we have

$$DF^2 = CD^2 + CF^2 - 2CF \cdot CD \cos 135°$$
$$2 = 1 + x^2 + \sqrt{2}x$$
$$x = \frac{\sqrt{3} - 1}{\sqrt{2}}$$

Let the $\angle DFC = \theta$. Again, by Cosine formula,

$$CD^2 = DF^2 + CF^2 - 2DF \cdot CF \cos\theta$$
$$1 = 2 + \left(\frac{\sqrt{3} - 1}{\sqrt{2}}\right)^2 - 2\sqrt{2}\left(\frac{\sqrt{3} - 1}{\sqrt{2}}\right)\cos\theta$$
$$\cos\theta = \frac{\sqrt{3}}{2}$$

Hence $\angle DFC = 30°$ and $\angle CDF = 45° - \angle DFC = 15°$. Answer A.

PART – B

16. $ABCD$ is a square. E, F are points on BC, CD respectively and $\angle EAF = 45°$. The value of $\dfrac{EF}{BE + DF}$ is ———

17. The average of 5 consecutive natural numbers is 10. The sum of the second and fourth of these numbers is ———

Solution The numbers can be assumed as $m-2, m-1, m, m+1, m+2$. Given that their average $m = 10$. Thus the numbers are $8, 9, 10, 11, 12$ and the sum of second and fourth of these numbers is $2m = 20$.

18. The number of natural numbers n for which $n^2 + 96$ is a perfect square is ———

Solution Let $m^2 = n^2 + 96$, where m is a natural number. Then $m^2 - n^2 = 96$ and $(m-n)(m+n) = 96$. Thus

$$m - n = 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96$$

Since $m + n > m - n$, it follows that $m - n = 1, 2, 3, 4, 6, 8$ and the corresponding values of $m + n$ are $96, 48, 32, 24, 16, 12$. Solving these, the possible values for n are $23, 10, 5, 2$. Hence there are four such natural numbers.

19. n is an integer and $\sqrt{\dfrac{3n-5}{n+1}}$ is also an integer. The sum of all such n is ———

Solution Since $\dfrac{3n-5}{n+1} = \dfrac{3(n+1)-8}{n+1} = 3 - \dfrac{8}{n+1}$, it follows that $n+1$ must be a power of 2. Hence $n = \pm 1, \pm 2, \pm 4, \pm 8$. Of these only when $n+1 = 4$ or -8, we get $\dfrac{3n-5}{n+1}$ as a perfect square of a natural number. Hence $n = 3$ or -9 and the sum of these values is -6.

20. $\dfrac{a}{b}$ is a fraction where a, b have no common factors other than 1. b exceeds a by 3. If the numerator is increased by 7, the fraction is increased by unity. The value of $a+b$ ———

Solution $b = a + 3$ and $\dfrac{a+7}{b} = \dfrac{a}{b} + 1$. Hence $\dfrac{7}{b} = 1$ and consequently $b = 7$ and $a = 4$. Thus $a + b$ is 11.

21. If $x = \sqrt[3]{2} + \dfrac{1}{\sqrt[3]{2}}$, then the value of $2x^3 - 6x$ is ———

Solution We have

$$x^3 = 2 + \frac{1}{2} + 3\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{2}}\right) = \frac{5}{2} + 3x$$

Hence $2x^3 - 6x = 5$.

22. The angles of a heptagon are

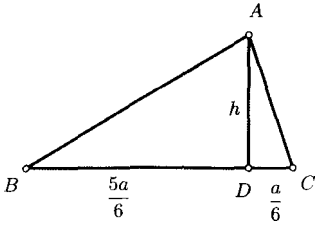
$$160°, 135°, 185°, 140°, 125°, x°, x°$$

The value of x is ———

Solution The sum of the angles of a heptagon is $(7-2)\times 180° = 900°$. Hence $60°+135°+185°+140°+125°+2x° = 900°$ and $x = 77.5°$.

23. ABC is a triangle and AD is its altitude. If $BD = 5DC$, then the value of $\dfrac{3(AB^2 - AC^2)}{BC^2}$ is ———

Solution Since $BD+DC = a$, it follows that $BD = \dfrac{5a}{6}$ and $DC = \dfrac{a}{6}$. Let $AD = h$.



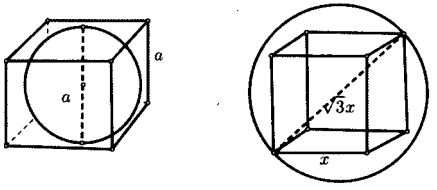
$$\frac{3(AB^2 - AC^2)}{BC^2} = \frac{3\left((h^2 + (5a/6)^2) - (h^2 + (a/6)^2)\right)}{a^2}$$

$$= \frac{2a^2}{a^2} = 2$$

24. A sphere is inscribed in a cube that has a surface area of $24\ \text{cm}^2$ such that its surface touches all the six faces of the cube. Inside the sphere, the largest possible cube is incribed. The surface area of the inner cube (in cm^2) is

---

Solution



The surface area of a cube is $6a^2$, where a is the length of an edge. The given sphere, hence has edge length 2. This equals the diameter of the inner sphere and when we inscribe a second cube inside this sphere, the length of the diagonal of the cube equals the diameter of the sphere. Thus if x denotes the edge of this second cube, then $\sqrt{3}x = 2$. Thus the surface area is $6x^2 = 8\ \text{cm}^2$.

25. A positive integer n is a multiple of 7. If $\sqrt{n}$ lies between 15 and 16, the number of possible value(s) of n is ———
Solution We have $225 < n < 256$. The multiples of 7 that lie between 225 and 256 are $231, 238, 245, 252$ and hence the number of multiples is 4.

26. The value of x which satisfies the equation

$$\frac{\sqrt{x+5}+\sqrt{x-16}}{\sqrt{x+5}-\sqrt{x-16}} = \frac{7}{3}$$

is ———
Solution We know that if $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$.

Hence we have

$$\frac{\sqrt{x+5}}{\sqrt{x-16}} = \frac{5}{2}$$

Squaring and simplifying, we get $4(x+5) = 25(x-16)$ and hence $x = 20$.

27. M man do a work in m days. If there had been N men more, the work would have been finished n days earlier, then the value of $\frac{m}{n} - \frac{M}{N}$ is ———

Solution If d is the amount of work done by one man in a day, then we have

$$\begin{aligned} & Mmd = (M+N)(m-n)d \\ \Rightarrow & \frac{m}{m-n} = \frac{M+N}{M} \\ \Rightarrow & mN - nM = Nn \\ \Rightarrow & \frac{m}{n} - \frac{M}{N} = 1 \end{aligned}$$

28. The sum of the digits of a two digit number is 15. If the digits of the given number are reversed, the number is increased by the square of 3. The original number is ———

Solution Let the number be $10a+b$. We have $a+b = 15$ and $(10b+a)-(10a+b) = 9$ and hence $b-a = 1$. Hence $a = 7$ and $b = 8$ and the number is 78.

29. When expanded the units place of $(3127)^{173}$ is ———

Solution The units digit of $(3127)^{173}$ depends only on the units digit of 3127. Hence we need to find the units digit of 7^{173}. The units digit of consecutive powers of 7 are $7, 9, 3, 1, 7, 9, 3, 1, \ldots$. Since $173 = 1 \mod 4$, it follows that the units digit is 7.

30. If $a : (b+c) = 1 : 3$ and $c : (a+b) = 5 : 7$, then $b : (c+a)$ is ———

Solution Since $\frac{a}{b+c} = \frac{1}{3}$, we have $\frac{a+b+c}{b+c} = \frac{4}{3}$ and hence $3(a+b+c) = 4(b+c)$. Similarly, from $\frac{c}{a+b} = \frac{5}{7}$, we get $7(a+b+c) = 12(a+b)$. Thus

$$\begin{aligned} 12(b+c) + 12(a+b) &= 9(a+b+c) + 7(a+b+c) \\ &= 16(a+b+c) \\ \Rightarrow 12b &= 4(a+b+c) \\ \Rightarrow a+c &= 2b \end{aligned}$$

Thus $b : (c+a) = 1 : 2$.

SCREENING TEST – RAMANUJAN CONTEST
NMTC at INTER LEVEL
XI & XII Standards

PART – A

1. In the addition shown, each of the letters T, H, I, S represents a non zero digit. What is $T + H + I + S$?

$$\begin{array}{cccc} T & H & I & S \\ & & I & S \\ \hline 2 & 0 & 1 & 8 \end{array}$$

A. 34 B. 32 C. 24 D. 22

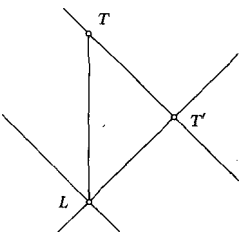
Solution The sum of S and S has units digit 8. Hence $S = 4$ or 9. If $S = 4$, then the sum would not have a carry and $I + I$ would be even. But the tenths digit of the sum is odd and hence $S = 9$. Since $I + I + 1$ ends with 1, it follows that $I = 5$. This forces $H = 9$ and $T = 1$. Thus $T+H+I+S = 1+9+5+9 = 24$. Answer C.

2. We have four sets S_1, S_2, S_3, S_4 each containing a number of parallel lines. The set S_i contains $i + 1$ parallel lines, $i = 1, 2, 3, 4$. A line in S_i is not parallel to lines in S_j when $i \neq j$. In how many points do these lines intersect?

A. 54 B. 63 C. 71 D. 95

Solution When a set of p parallel lines intersects a set of q parallel lines, each line of the first set meets each line of the second set and so there are $p \times q$ points of intersection. Here there are 4 sets of parallel lines, and there are 6 pairs of these sets that intersect each other. The total number of intersections is $2\times3+2\times4+2\times5+3\times4+3\times5+4\times5 = 71$. Answer C.

3. An oil tanker is 100 km due north of a cruise liner. The tanker sails Southeast at a speed of 20 kilometers per hour and the liner sails Northwest at a speed of 10 kilometers per hour. What is the shortest distance between the two boats during the subsequent motion?
   A. $50\sqrt{2}$ km B. 60 km C. 80 km D. 100 km
   Solution The initial positions of the oil tanker, T, and



   the cruise liner, L, are as shown in the figure. Since they travel in parallel directions, the relative motion of the tanker is 10 km per hour in the Northwest direction. Thus the shortest distance happens when LT' is perpendicular to the Northwest direction and hence the shortest distance is $LT\cos 45° = 50\sqrt{2}$ kms. Answer A.

4. Volume A equals one fourth of the sum of the volumes B and C, while volume B equals one sixth of the sum of the volumes A and C. The ratio of sum of volumes A and B C to volume C is
   A. 2 : 3 B. 9 : 10 C. 7 : 12 D. 12 : 23
   Solution Given $A = \frac{1}{4}(B + C), B = \frac{1}{6}(A + C)$. Hence $4A - B = C$ and $-A + 6B = C$. Solving, we get $A = \frac{7}{23}C$ and $B = \frac{5}{23}C$. Hence $A + B = \frac{12}{23}C$ and $(A + B) : C = 12 : 23$. Answer D.

5. In the *ninety-nine* shop every item costs some whole

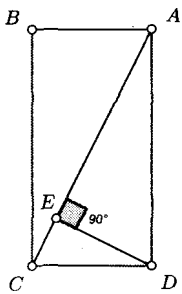
number of rupees plus 99 paise. Rhea spent sixty five rupees and seventy six paise in buying some items from the shop. How many items did she buy?

A. 23 B. 24 C. 65 D. 66

Solution Let k be the number of items she bought. The total money spent by her would be a whole number of rupees plus $99k = (100 - 1)k$ paise and hence a whole number of rupees less k paise. Since she spent 65 rupees and 76 paise, which can be written as 66 rupees less 24 paise, the possible values for k are $24, 124, 224, \ldots$. Since each item costs at least 99 paise, the number of items bought by her is at most 66. Hence $k = 24$. Answer B.

6. The diagram shows a rectangle $ABCD$ where $AB : AD = 1 : 2$. Point E on AC is such that DE is perpendicular to AC. What is the ratio of the area of the triangle DCE to the rectangle $ABCD$?

A. $1 : 4\sqrt{2}$ B. $1 : 6$ C. $1 : 8$ D. $1 : 10$



Solution We can assume that $AB = 1$. Then $AD = 2$, $AC = \sqrt{2^2 + 1^2} = \sqrt{5}$. Triangles CED and ADC are similar. Thus $\frac{DE}{AD} = \frac{CD}{AC} = \frac{CE}{CD}$. Hence $\frac{DE}{2} =$

$\frac{1}{\sqrt{5}} = \frac{CE}{1}$. Thus $CE = \frac{1}{\sqrt{5}}$ and $DE = \frac{2}{\sqrt{5}}$. Area of $CED = \frac{1}{2}CE \cdot DE = \frac{1}{5}$. Since the area of $ABCD$ is 2, the ratio is $1 : 10$. Answer D.

7. The numbers $2, 3, 12, 14, 15, 20, 21$ may be divided into two sets so that the product of the numbers in each set is the same. What is this product?

   A. 420 B. 1260 C. 2520 D. 6720

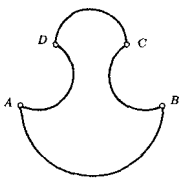
   Solution If we multiply all the given numbers, then the product will be the square of the product of the numbers in each of the sets. Since

   $$2 \times 3 \times 12 \times 14 \times 15 \times 20 \times 21 = (2^3 \times 3^2 \times 5 \times 7)^2$$

   it follows that the product of numbers in each set is $2^3 \times 3^2 \times 5 \times 7 = 2520$. Answer C.

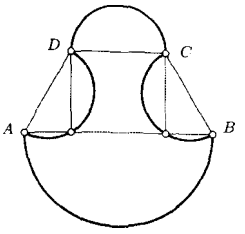
8. $ABCD$ is a trapezium with $AD = DC = CB = 10$ units and $AB = 22$ units. Semi circles are drawn as shown in the figure. The area of the region bounded by these semi circles in square units is

   A. $128 + 48\pi$ B. $128 + 24\pi$ C. $116 + 48\pi$



   D. $116 + 24\pi$

   Solution The required area is composed of the area of the trapezium $ABCD$ plus the areas of the semi circles on AB and CD less the area of the semi circles on AD



and BC. Since $AB = 22$ and $CD = 10$, it follows that $BC = 6$ and the height of the trapezium is $\sqrt{10^2 - 6^2} = 8$. Area of the trapezium is $\frac{1}{2} \times 8 \times (22 + 10) = 128$ and the sum of treas of the semi circles on CD and AB is $\frac{1}{2}(121\pi + 25\pi) = 73\pi$. Sum of areas of semicircles on BC and AD is 25π. Hence the required area of the region is $128 + 73\pi - 25\pi = 128 + 48\pi$. Answer A.

9. Consider the number of ways in which five girls and five boys sit in ten seats that are equally spaced around a circle. The proportion of the seating arrangements in which no two girls sit at the ends of a diameter is

A. $\frac{1}{2}$ B. $\frac{8}{63}$ C. $\frac{55}{63}$ D. None of the above

Solution There are five diameters and place one girl in each of the diameters. This can be done in $2^5 = 32$ ways. The total number of ways of choosing the five seats for the girls is $\binom{10}{5} = 252$. Hence the proportion of seating arrangements in which no two girls sit at the ends of a diameter is $\frac{32}{252} = \frac{8}{63}$. Answer B.

10. Let $A = 1^{-4} + 2^{-4} + 3^{-4} + \cdots$, the sum of reciprocals of fourth powers of integers and $B = 1^{-4} + 3^{-4} + 5^{-4} + \cdots$, the sum of reciprocals of fourth powers of odd positive

integers. The value of A/B as a fraction is

A. $\frac{16}{15}$ B. $\frac{32}{31}$ C. $\frac{64}{63}$ D. $\frac{128}{127}$

Solution

$$\begin{aligned} B &= \left(1^{-4}+2^{-4}+3^{-4}+\cdots\right)-\left(2^{-4}+4^{-4}+6^{-4}+\cdots\right) \\ &= A-\frac{1}{16}\left(1^{-4}+2^{-4}+3^{-4}+\cdots\right) \\ &= A-\frac{1}{16}A=\frac{15}{16}A \end{aligned}$$

Hence $A/B = 16/15$. Answer A.

11. The number $5^{(6^7)}$ is written on the board (in base 10). Gia takes two of the digits at a time, erases them but appends the sum of those digits at the end. She repeats this till she ends up with one digit on the board. What is the digit that remains on the board?

A. 1 B. 5 C. 6 D. 7

Solution Suppose a, b are two of the digits. When we remove them but append their sum at the end, the sum of the digits on the board either remains the same (for example when we have a, b such that $a + b \leq 9$) or reduces by 9 (when $a + b > 9$, then we append $a + b - 9$ at the end – for example, if $a = 8, b = 7$, the original sum is 15 but we append the number 15 and hence the sum of the digits become $6 = 15 - 9$).

Hence when we are left with one digit, that would be the remainder when the given number is divided by 9.

The remainder when consecutive powers of 5 are divided by 9 are $5, 7, 8, 4, 2, 1, 5, 7, \ldots$. Since 6^7 is a multiple of 6, it follows that the digit left on the board is 1. Answer A.

12. Seven points are marked on the circumference of a circle and all pairs of points are joined by straight lines. No three of these lines have a common point and any two intersect at a point inside the circle. Into how many regions is the interior of the circle divided by these lines?
A. 64 B. 63 C. 57 D. 56

Solution We first prove: The number of regions determined by L lines, no three concurrent inside the circle, which produce P points of intersection in the circle is $P+L+1$.

Suppose we delete the lines one at a time. Let us count the number of regions lost by the deletions. Each section of a line separates two regions which unite into a single region when the line is deleted. The number of regions lost is therefore one more than the number of intersection points on the line. Now each point of intersection lies on two lines and the intersection point is removed from both the lines when we delete a line. Thus each point of intersection figures exactly once in this process of deletion. Thus for each line, the number of regions lost is the number of points of intersection left on the line plus 1. Summing over the deletion of the L lines, we see that our total will contain all the P points of intersection plus a 1 for each line, implying that a total of $P+L$ regions are lost. Since the interior of the circle still remains after all deletions, the total number of regions started with must be $P+L+1$.

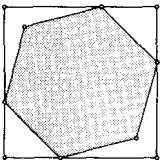
When there are n points on the circle, they determine $\binom{n}{2}$ chords and each point of intersection is determined by 4 points. Hence the number of points of intersection is $\binom{n}{4}$. The total number of regions is, therefore,

$1 + \binom{n}{2} + \binom{n}{4}$. For $n = 7$, the number of regions is $1 + \binom{7}{2} + \binom{7}{4} = 57$.

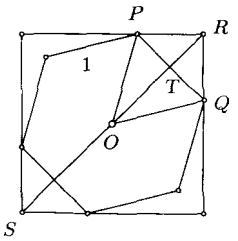
For another proof of this result, see S.Muralidharan, *Combinatorics (For Mathematical Olympiads)*, (AIMER Publications), Chapter 1, Section 13.

13. The diagram below shows a regular hexagon with side length 1, inscribed in a square. Two of the vertices lie on the diagonal of the square and the remaining vertices lie on its sides. What is the area of the square?

A. $\frac{7}{2}$ B. 4 C. $2+\sqrt{3}$ D. $3+\sqrt{2}$



Solution Let P, Q be two of the adjacent vertices of the hexagon that lie on the sides of the square. Let O be



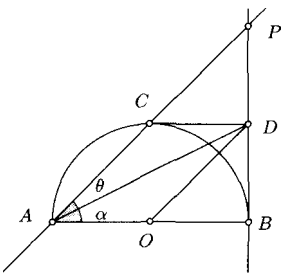
the center of the square. Since OPQ is an equilateral

triangle, it follows that $QT = \frac{1}{2}$ and $OT = \frac{\sqrt{3}}{2}$. Also, $RT = TQ = \frac{1}{2}$. Hence the diagonal $SR = 2OR = 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = \sqrt{3} + 1$. Hence the area of the square is $\frac{1}{2}SR^2 = 2 + \sqrt{3}$. Answer C.

14. AB is a diameter of a semicircle of center O. C is the midpoint of the arc AB. AC and the tangent at B to the semicircle meet at P. D is the midpoint of BP. If $ACDO$ is a parallelogram and $\angle PAD = \theta$, then $\sin\theta$ is

A. $\frac{1}{\sqrt{5}}$ B. $\frac{1}{\sqrt{10}}$ C. $\frac{2}{\sqrt{10}}$ D. $\frac{3}{\sqrt{10}}$

Solution Clearly, if r is the radius of the circle, $DB =$



r. Let $\angle DAB = \alpha$. Then $\tan\alpha = \frac{r}{2r} = \frac{1}{2}$. Since

$\angle CAO = 45°$, we have

$$\begin{aligned}\sin\theta &= \sin(45° - \alpha)\\ &= \frac{1}{\sqrt{2}}(\cos\alpha - \sin\alpha)\\ &= \frac{1}{\sqrt{2}}\left(\frac{2}{\sqrt{5}} - \frac{1}{\sqrt{5}}\right)\\ &= \frac{1}{\sqrt{10}}\end{aligned}$$

15. The real valued function $f(x)$ satisfies the equation $2f(1-x)+1 = xf(x)$ for all x. Then $(x^2 - x + 4)f(x)$ equals

A. $x-1$ B. x C. $x+1$ D. $x-3$

Solution Replacing x by $1-x$ in the given equation, we get $2f(x)+1 = (1-x)f(1-x)$. Using $2f(1-x)+1 = xf(x)$ and solving for $f(x)$, we get $(x^2 - x + 4)f(x) = x + 1$. Answer C.

PART – B

16. The number of ways in which 26 identical chocolates be distributed between Amy, Bob, Cathy and Daniel so that each receives at least one chocolate and Amy receives more chocolates than Bob is ———

Solution Let x_1, x_2, x_3 and x_4 be the number of chocolates received by Amy Bob, Cathy and Daniel respectively. Then $x_i \geq 1$, $x_1 > x_2$ and $x_1 + x_2 + x_3 + x_4 = 26$.

The number of solutions in positive integers to the equation $x_1 + x_2 + x_3 + x_4 = 26$ is $\binom{25}{3} = 2300$. Of these, the number of solutions in which $x_1 = x_2$ is same as the number of solutions to $2x + x_3 + x_4 = 26$ in positive

integers. The possible values for x are $1, 2, \ldots, 12$. When $x = i$, the number of solutions is $26-2i-1$. Hence the number of solutions is $23 + 21 + \cdots + 1 = 144$. Now, the number of solutions to the equation $x_1+x_2+x_3+x_4 = 26$ such that $x_1 \neq x_2$ is $2300-144 = 2156$. Of these half of them would have $x_1 > x_2$. Thus the required number of ways of distributing the chocolates is $\frac{2156}{2} = 1078$.

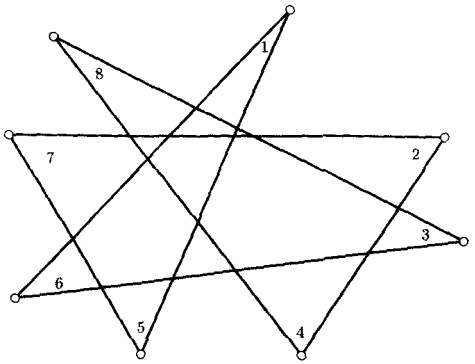
17. A set S contains 11 numbers. The average of the numbers in S is 202. The average of the six smallest numbers of S is 100 and the average of the six largest of the numbers is 300. What is the median of the numbers in S ———

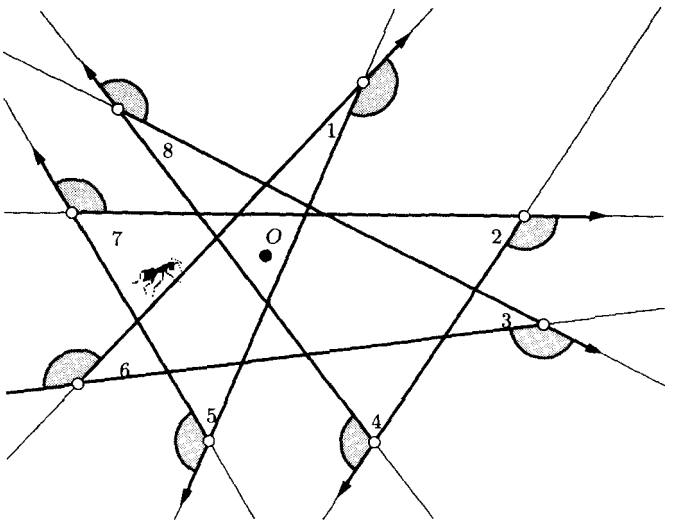
Solution The total of the 11 numbers is $11 \times 202 = 2222$. Sum of the smallest six numbers is $s = 600$ and sum of the largest six numbers is $S = 1800$. If we add $s+S$, we will be adding the median m twice in the sum. Hence sum of the numbers plus the median m equals $600 + 1800 = 2400$. Since the sum of the numbers is 2222, it follows that $m = 2400 - 2222 = 178$. Since $100 < 172 < 300$, such a set of 11 numbers is possible.

18. The sum of the angles $1, 2, 3, 4, 5, 6, 7, 8$ in degrees shown in the following figure is ———

Solution Consider an ant traversing the polygon along the lines. When it reaches a vertex, it changes the direction by $180^\circ - \theta$ where θ is the angle at that vertex. Thus it changes the direction by $\sum_{i=1}^{8}(180^\circ - \angle i)$. Noting that in returning to the starting point, it would have moved around the point O three times, it follows that $\sum_{i=1}^{8}(180^\circ - \angle i) = 3 \times 360^\circ = 1080^\circ$. Hence $\sum \angle i = 8 \times 180^\circ - 1080^\circ = 360^\circ$.

19. The number of positive integers less than 2018 that are





divisible by 6 but are not divisible by at least one of the numbers 4 or 9 is ———

Solution There are $\left\lfloor \frac{2018}{6} \right\rfloor = 336$ positive integers that are less than 2018 that are divisible by 6. Let A be the set of positive integers less than 2018 that are divisible by $6 \times 2 = 12$ and B, the set of positive integers less

than 2018 that are divisible by $6 \times 3 = 18$. Then

$$\begin{aligned} |A \cup B| &= |A| + |B| - |A \cap B| \\ &= \left\lfloor \frac{336}{2} \right\rfloor + \left\lfloor \frac{336}{3} \right\rfloor - \left\lfloor \frac{336}{6} \right\rfloor \\ &= 168 + 112 - 56 = 224 \end{aligned}$$

Thus, the number of positive integers less than 2018 divisible by 6 but not divisible by 4 or 9 is $336 - 224 = 112$.

20. If

$$x(x+1)(x+2)\cdots(x+23) = \sum_{n=1}^{24} a_n x^n$$

the number of coefficients a_n that are multiples of 3 is

———

Solution The given product modulo 3 is

$$\begin{aligned} x^8(x+1)^8(x+2)^8 &= x^8(x+1)^8(x-1)^8 \pmod 3 \\ &= x^8(x^2-1)^8 \pmod 3 \end{aligned}$$

Since $\binom{8}{r}$ is not a multiple of 3 for $0 \le r \le 8$, it follows that the number of coefficients that are multiples of 3 are $24 - 9 = 15$.

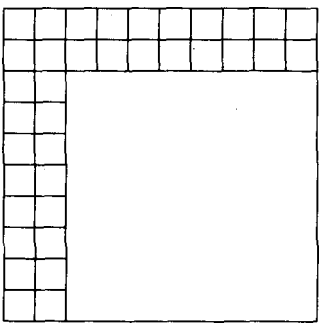
21. A square is cut into 37 squares of which 36 have area 1 square cms. The length of the side of the original square is ———

Solution If the 37th square together with the thirty six 1×1 1squares, makes up a square, both the 37th square, and the original square must have side lengths which are an integer number of cms. So, if the side of the original

square is x cm, we require that $x^2 - 36$ is also the square of an integer. Hence

$$y^2 = x^2 - 36 \Leftrightarrow x^2 - y^2 = 36 \Leftrightarrow (x+y)(x-y) = 36$$

Since $x+y$ and $x-y$ have the same parity, $x+y$ and $x-y$ both must be even. The only possible solution is $x+y = 18$ and $x-y = 2$. Hence $x = 10$.
Finally, we need to check that 36 squares of area 1 square cm, can be fitted together with a square of side length 8, to make a square of side length 10. This is shown in the Figure below.



22. There are 4 coins in a row and all are showing heads to start with. The coins can be flipped with the following rules:

    (a) The fourth coin (from the left) can be flipped any time

    (b) An intermediate coin can be changed to tail only if its immediate neighbor on the right is heads and all other coins (if any) to its right are tails.

(c) Only one coin can be flipped in one step.

The minimum number of steps required to bring all coins to show tails is ———

23. A poet met a tortoise sitting under a tree. When the tortoise was the poet's age, the poet was only a quarter of his current age. When the tree was the tortoise's age, the tortoise was only a seventh of its current age. If all the ages are in whole number of years, and the sum of their ages is now 264, the age of the tree in years is ———

Solution Let the ages of the poet, tortoise and tree in years be x, y, z respectively. We have

$$x-(y-x)=\frac{x}{4}, \quad y-(z-y)=\frac{y}{7}, \quad x+y+z=264$$

From the first two equations, we get $7x = 4y$ and $13y = 7z$. Hence $x = \frac{z}{13}$ and $y = \frac{7z}{13}$. Substituting in the third equation, we obtain

$$\frac{4z}{13}+\frac{7z}{13}+z=264$$

and $z = 143$.

24. The sum of all real values of x satisfying

$$\left(x+\frac{1}{x}-17\right)^2 = x+\frac{1}{x}+17$$

is ———

Solution Sum of the roots of the equation is equal to the $-a$, where a is the coefficient of x^3 when we expand the expression as a polynomial equation in x. Now,

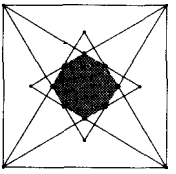
$$\left(x+\frac{1}{x}-17\right)^2-\left(x+\frac{1}{x}\right)-17=0$$

$$(x^2-17x+1)^2-x(x^2+17x+1)=0$$

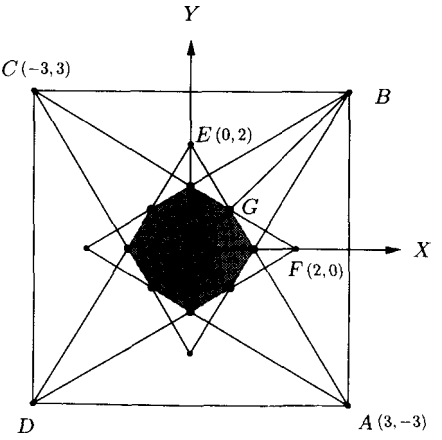
and hence the given equation is $x^4 - 35x^3 + \cdots = 0$ and sum of the roots is 35.

25. On the inside of a square with side length 6, construct four congruent isosceles triangles each with base 6 and height 5, and each having one side coinciding with a different side of the square. The area of the octagonal region common to the interiors of all four triangles is ———

Solution Let the vertices of the square be A, B, C and



D and E, F, G, H points as shown in the figure below. Taking the center of the square as the origin, the equation



of the line CF is $3x + 5y = 6$. Hence the coordinates of the point H is $\left(0, \frac{6}{5}\right)$. Since the point G is the intersection of $x = y$ and the line CF, the coordinates of the point G is $\left(\frac{3}{4}, \frac{3}{4}\right)$. Thus the area of the triangle OHG is $\frac{18}{40}$. Since the area of the octagon is 8 times the area of OHG, it follows that its area is $\frac{18}{5}$.

26. In a triangle with integer side lengths, one side is thrice the other. The third side is 15 cm. The greatest possible perimeter of the triangle is (in cm) ———

Solution Let two of the sides of the triangle be $x, 3x$. We have $4x > 15$ and $x + 15 > 3x$. Thus $4 < x < 8$. Thus the greatest possible length of the sides is 21 and the perimeter of the triangle is $7 + 21 + 15 = 43$.

27. A cube has edge length x (an integer). Three faces meeting at a corner are painted blue. The cube is then cut into smaller cubes of unit length. If exactly 343 of these cubes have no faces painted blue, then the value of x is ———

Solution The number of smaller cubes that have no faces painted is $(x-1)^3$. Hence $(x-1)^3 = 343 = 7^3$ and $x = 8$.

28. If $f(x) = ax^4 - bx^2 + x + 5$ and $f(3) = 8$, the value of $f(-3)$ is ———

Solution We have

$$\begin{aligned} f(-3) &= a(-3)^4 - b(-3)^2 - 3 + 5 \\ &= a3^4 - b3^2 - 3 + 5 \\ &= (a3^4 - b3^2 + 3 + 5) - 6 \\ &= f(3) - 6 = 8 - 6 = 2 \end{aligned}$$

29. Archana has to choose a three-digit code for her bike lock. The digits can be chosen from 1 to 9. To help her remember them, she decides to choose three different digits in increasing order, for example 278. The number of such codes she can choose is ———

Solution We can choose three distinct digits from 1 to 9 in $\binom{9}{3} = 84$ ways and for each such choice of three digits we get one three digit code in increasing order. Thus the number of three digit codes is 84.

30. Let S be a set of five different positive integers, the largest of which is n. It is impossible to construct a quadrilateral with non-zero area, whose side-lengths are all distinct elements of S. The smallest possible value of n is ———

Solution Let $ABCD$ be a quadrilateral, with non-zero area. We note first that the shortest route from A to D is the line segment AD. Since $ABCD$ has non-zero area, the points B and C are not both on the line segment AD. Therefore the path made up of the line segments AB, BC and CD is not the shortest route from A to D. It follows that $AB + BC + CD > AD$. Conversely, it may be seen that when the sum of the three smallest of four positive numbers is greater than the largest number of the four, then there is quadrilateral with non-zero area which has these four positive numbers as its side lengths. Suppose h, j, k, l and m are five positive integers with $h < j < k < l < m$, which have the required property that, whichever four of them are selected, these are not the side lengths of a quadrilateral with non-zero area. Then, whichever four of these numbers are selected, the sum of the three smallest is no greater than the largest.

It is not difficult to show that if $h + j + k \leq l$ and $j + k + l \leq m$, the integers h, j, k, l and m satisfy the condition that, whichever four of them are selected, the sum of the three smallest is no greater than the largest. We wish to choose m to be as small as possible. To achieve this we need h, j, k and l to be as small as possible. The smallest possible values for h, j and k, are $h = 1, j = 2$ and $k = 3$. This gives $h+j+k = 1+2+3 = 6$. Because we need to have $h + j + k \leq l$, the smallest possible value of l is 6. Then $j + k + l = 2 + 3 + 6 = 11$. Because we need to have $j + k + l \leq m$, the smallest possible value of m is 11.

FINAL – GAUSS CONTEST
NMTC at PRIMARY LEVEL
V & VI Standards

1. Write down all the ten digit numbers whose digit sum is 2 (the digit sum of a number is the sum of its digits – for example, the digit sum of 4022 is $4+0+2+2=8$). Find the sum of all ten digit numbers whose digit sum is 2.
Solution The leading digit is either 2 or 1. If the leading digit is 2, all other digits must be 0 and hence there is only one such number – 2000000000. If the leading digit is 1, there are 9 such numbers:

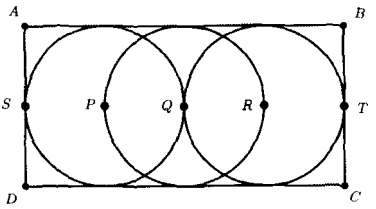
   1100000000, 1010000000, 1001000000, 1000100000, 1000010000, 1000001000, 1000000100, 1000000010, 1000000001

   The sum of all such numbers is 11111111111.

2. The sum of three digit numbers $35a$ and $4b7$ is divisible by 36. Find all possible pairs (a, b).
Solution A number is divisible by 36 if and only if it is divisible by 9 and 4. A number is divisible by 9 if and only if the sum of its digits is divisible by 9. Since $35a+4b7$ is divisible by 9, it follows that $3+5+a+4+b+7 = 19+a+b$ must be divisible by 9. Hence $1+a+b$ is divisible by 9. Since both a, b are st most 9, it follows that $a+b = 8$ or 17. Since the units digit of the sum is the units digit of $a+7$, it follows that a must be odd. If $a+b = 17$, the only possibility is $a = 9, b = 8$. In this case $35a + 4b7 = 359 + 487 = 846$ and this is not a multiple of 4. Hence $a+b = 8$. The possibilities for (a, b) are $(1,7), (3,5), (5,3)$ and $(7,1)$. In these

cases, the sums are respectively, $828, 810, 772, 774$. Of these only 828 and 772 are multiples of 4. Thus the only possibilities are $(a, b) = (1, 7)$ or $(5, 3)$.

3. Three congruent circles with centers P, Q, R are tangent to the sides of rectangle $ABCD$ as shown. The circle with center Q has diameter 5 cm and passes through the points P and R. Find the area of the rectangle $ABCD$.
Solution BC equals the diameter of the circles and

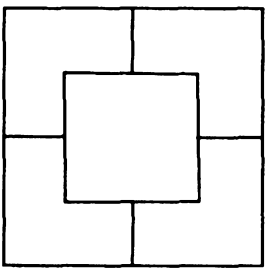


hence equals 5 cm. Since the circles have equal diameters, $AB = PR + SP + RT = 10$ cm. Hence the area is 50 square cms.

4. A lucky year is one in which, at least one date, written in the form day / month / year, has the following property: the product of the month and day equals the last two digits of the year. For example, 1944 is a lucky year since it has the date $11/4/44$. From 1951 to 2000 how many years are not lucky? Give proper explanation for your answer.
Solution Clearly if the last two digits of the year is a prime number greater than 31, then the year is not lucky. Thus the years $1953, 1959, 1961, 1967, 1971, 1973, 1979, 1983, 1989$, and 1997 are not lucky. 1958 is not lucky since $58 = 2 \times 29$ and 1958 being a non leap

year has only 28 days in February. For a similar reason, 1962, 1974, 1982, 1986, 1994 are not lucky. Clearly 2000 is not lucky. It is not difficult to see other years in the range 1951 to 2000 are lucky. Hence the number of not lucky years is 17.

5. The area of each of the four congruent L-shaped regions in this 100 cms by 100 cms square is 3/16 of the total area. What is the length of the side of the center square?
Solution The total area of the L-shaped regions is



$4 \times \frac{3}{16} = \frac{3}{4}$ of the area of the square. Hence the area of the center square is $\frac{1}{4} \times 10000 = 2500$ square cms. Thus its side is 50 cms.

6. For any positive integer n, let $s(n)$ denote the sum of the digits of n. What is the minimum value of $\frac{n}{s(n)}$ where (1) $10 \leq n \leq 99$, (2) $100 \leq n \leq 999$?
Solution (1) For $10 \leq n \leq 19$, $\frac{n}{s(n)}$ decreases from $\frac{10}{1} = 10$ to $\frac{19}{10} = 1.9$. For $20 \leq 30$, $\frac{n}{s(n)}$ decreases from $\frac{20}{2} = 10$ to $\frac{29}{11} = 2\frac{7}{11}$. Similarly, in the other

intervals 30 to 39, 40 to 49, ..., 90 to 99, the minimum of the fraction $\frac{n}{s(n)}$ are $\frac{39}{12}, \frac{49}{13}, \ldots, \frac{99}{18}$. Of these, clearly, $\frac{19}{10}$ is the minimum.

(2) The least value of the fraction $\frac{n}{s(n)}$ when $100 \leq n \leq 199$ is attained at $n = 199$ with value $\frac{199}{19}$. For $200 \leq n \leq 999$, the value of the fraction increases. Hence the minimum value is attained at $n = 199$.

7. A 122 digit number is obtained by writing the 2 digit numbers 39 to 99, that is $394041 \ldots 9899$. You have to remove 61 digits from this number such that the remaining digits in that order form the largest number possible. (For example, in the number 15161718, if we remove the four 1s, we get the number 5678, but if we remove $1, 5, 1$ and the 1 after 6, we get 6718. This will be the largest possible number in this case). What will be the first ten digits of the largest number obtained?

Solution For the number obtained after removing 61 digits to be the largest, it must have as many leading 9s as possible. Let us remove the first 3, retain the 9 and remove the digits as follows: We have a removed a total of $1 + 19 + 19 + 19 = 58$ digits. For further three digits, choose the $0, 1, 2$ from $707172 \ldots$ to get the number $9999777737475 \ldots 9899$. This is the largest number and hence the first ten digits are 9999777737.

8. Given the numbers $2, 4, 8, 10, 14, 16$. Define $a\%b$ to be the remainder when the product $a \times b$ is divided by 18. Find the % product of every pair of these numbers including the product of a number with itself and fill the table given below:

%	2	4	8	10	14	16
2						
4						
8						
10						
14						
16						

(a) Find $2\%2\%\cdots\%2$, % product of fifteen 2s.

(b) Find $8\%8\%\cdots\%8$, % product of ten 8s.

Solution The table can be easily filled as follows:

%	2	4	8	10	14	16
2	4	8	16	2	10	14
4	8	16	14	4	2	10
8	16	14	10	8	4	2
10	2	4	8	10	14	16
14	10	2	4	14	16	8
16	14	10	2	16	8	4

(a) We have

$$2\%2\%2\%2\%2 = 14$$

Hence

$$\underbrace{2\%2\%\cdots\%2}_{15 \text{ times}} = 14\%14\%14 = 16\%14 = 8$$

(b) Since

$$8\%8\%8\%8\%8 = 10$$

it follows that $\underbrace{8\%8\%\cdots\%8}_{10 \text{ times}} = 10\%10 = 10$.

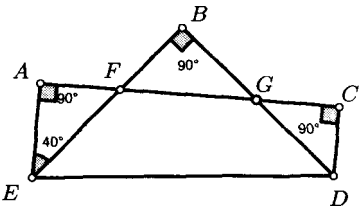
FINAL – KAPREKAR CONTEST
NMTC at SUB-JUNIOR LEVEL
VII & VIII Standards

1. A lucky year is one in which, at least one date, written in the form day / month / year, has the following property: the product of the month and day equals the last two digits of the year. For example, 1956 is a lucky year since for the date $7/8/1956$, we have $7 \times 8 = 56$. However 1962 is not a lucky year since $62 = 31 \times 2$ and hence there is no date in the year for which the required property holds. From 1900 to 2018 how many years are not lucky (not including 1900 or 2018)? Give proper explanation for your answer.

   Solution Clearly if the last two digits of the year is a prime number greater than 31, then the year is not lucky. Thus the years $1937, 1941, 1943, 1947$, $1953, 1959, 1961, 1967$, $1971, 1973, 1979, 1983, 1989$, and 1997 are not lucky. 1958 is not lucky since $58 = 2 \times 29$ and 1958 being a non leap year has only 28 days in February. For a similar reason, $1962, 1974, 1982, 1986, 1994$ are not lucky. Clearly 1900 and 2000 are not lucky. It is not difficult to see other years in the range 1900 to 2018 are lucky. Hence the number of unlucky years is 22.

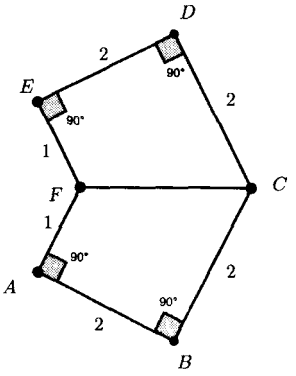
2. In the figure given, $\angle A, \angle B$ and $\angle C$ are right angles. If $\angle AEB = 40^\circ$ and $\angle BED = \angle BDE$, find $\angle CDE$.

   Solution From triangle AEF, we have $\angle AFE = 180^\circ - 90^\circ - 40^\circ = 50^\circ$. Now, $\angle BFG = \angle AFE = 50^\circ$, since they are vertically opposite angles. Again in triangle BFG, we have $\angle BGF = 180^\circ - 90^\circ - 50^\circ = 40^\circ$. Again, by vertically opposite angles, $\angle CGD = \angle FGB = 40^\circ$. Thus $\angle GDC = 50^\circ$. Since $\angle BED = \angle BDE = 45^\circ$, it follows that $\angle CDE = 50^\circ + 45^\circ = 95^\circ$.



3. (a) $ABCDEF$ is a hexagon in which $AB = BC = CD = DE = 2$ and $EF = FA = 1$. Its interior angle C is between 90° and 180° and $\angle F > 180°$. The rest of the angles are 90° each. What is its area?

   (b) A convex polygon with n sides has all its angles equal to 150° except one angle. List all possible values of n.



Solution

(a) Join CF. Since EF and CD are both

perpendicular to DE, $EF \parallel CD$. Thus $CDEF$ is a trapezium. Similarly, $ABCF$ is also a trapezium. Area of $CDEF = \frac{1}{2}DE \times (EF + CD) = 3$ and similarly, the area of $ABCF$ is also 3. Thus the area of the polygon is $3 + 3 = 6$ square units.

(b) Let the angle not equal to $150°$ be $x°$. Sum of the interior angles of the polygon is $(n-2)180° = (n-1)150° + x°$. Hence $30°(n-7) = x°$. Since the polygon is convex, $x° < 180°$ and $x \neq 150°$. Thus the possible values of n are $8, 9, 10, 11$.

4. If a, b, c are distinct non zero real numbers such that $\frac{1+a^3}{a} = \frac{1+b^3}{b} = \frac{1+c^3}{c}$ find all possible values of $a^3 + b^3 + c^3$.

Solution We have

$$\frac{1+a^3}{a} = \frac{1+b^3}{b} \Rightarrow \frac{1}{a} + a^2 = \frac{1}{b} + b^2$$
$$\Rightarrow \frac{1}{a} - \frac{1}{b} = b^2 - a^2$$
$$\Rightarrow \frac{b-a}{ab} = (b-a)(b+a)$$

Since $a \neq b$, we get $\frac{1}{ab} = a + b$. Similarly, $\frac{1}{bc} = b + c$. Subtracting, we get

$$\frac{1}{ab} - \frac{1}{bc} = a - c \Rightarrow abc = -1$$

Now, from $\frac{1}{ab} = a + b$, we get $-c = a + b$ and hence $a + b + c$. Thus

$$a^3 + b^3 + c^3 = 3abc = -3$$

5. Find the smallest positive integer such that it has exactly 100 positive integer divisors including 1 and itself.

Solution Any positive integer can be written as a product of powers of primes in a unique way. If

$$N = p_1^{a_1} p_2^{a_2} \cdots p_k^{a_k}$$

where $p_1, p_2, \ldots, p_k$ are distinct primes and $a_i > 0$ are integers, then N has $(a_1+1)(a_2+1)\cdots(a_k+1)$ divisors. Now, we need the smallest N having $100 = 2^2 \times 5^2$ divisors.

Let us look at an example to find the correct strategy. While $2^3 = 8$ has 4 divisors, a smaller number $2 \cdot 3$ also has four divisors. Hence to minimize the number with a given number of divisors, we can use more primes. Also, the numbers $2^2 \cdot 3 = 12$ and $2 \cdot 3^2 = 18$ both have 6 divisors but the number 12 is the smaller of the two. Hence we need to use the larger number with the smaller prime. Using this strategy, we find that the number $2^4 \cdot 3^4 \cdot 5 \cdot 7 = 45360$ has $5 \cdot 5 \cdot 2 \cdot 2 = 100$ divisors and this will be the smallest such number. Hence the required number is 45360.

6. (a) What is the sum of the digits of the smallest positive integer divisible by 99 and has all its digits equal to 2?

   (b) When 270 is divided by the odd number n, the quotient is a prime number and the remainder is 0. What is n?

Solution

(a) A number is divisible by 99 if and only if it is divisible by 9 and 11. Noting that a number is divisible by 9 if and only if the sum of its digits is divisible by 9, if a number with n digits all of whose digits are equal to 2 will be divisible by 9 if $2n$ is a multiple of 9. The values of n for which this is a multiple of 9 are $9, 18, \ldots$. The number with

nine 2s is not a multiple of 11 while the one with 18 twos is a multiple of 11. Hence the smallest number is the one which contains 18 twos. Thus the sum of the digits of the smallest number is $18 \times 2 = 36$.

(b) Since $270 = 3^3 \times 2 \times 5$, the odd divisor that gives quotient a prime number is $n = 3^3 \times 5 = 135$.

7. Consider the sums

$$A = \frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{99 \cdot 100}$$
$$B = \frac{1}{51 \cdot 100} + \frac{1}{52 \cdot 99} + \cdots + \frac{1}{100 \cdot 51}$$

Express $\frac{A}{B}$ as an irreducible fraction.

Solution

$$\begin{aligned} A &= \left(\frac{1}{1} - \frac{1}{2}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \cdots + \left(\frac{1}{99} - \frac{1}{100}\right) \\ &= \left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{100}\right) - 2\left(\frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{100}\right) \\ &= \frac{1}{51} + \frac{1}{52} + \cdots + \frac{1}{100} \end{aligned}$$

$$\begin{aligned} B &= \frac{1}{51 \cdot 100} + \frac{1}{52 \cdot 99} + \cdots + \frac{1}{100 \cdot 51} \\ &= \frac{1}{151}\left(\frac{151}{51 \cdot 100} + \frac{151}{52 \cdot 99} + \cdots + \frac{151}{100 \cdot 51}\right) \\ &= \frac{1}{151}\left(\frac{1}{51} + \frac{1}{100} + \frac{1}{52} + \frac{1}{99} + \cdots + \frac{1}{100} + \frac{1}{51}\right) \\ &= \frac{2}{151}\left(\frac{1}{51} + \frac{1}{52} + \cdots + \frac{1}{100}\right) \\ &= \frac{2}{151}A \end{aligned}$$

Thus $\frac{A}{B} = \frac{151}{2}$.

8. Let a, b, c be real numbers not all equal. Prove that if $a + b + c = 0$ then

$$a^2 + ab + b^2 = b^2 + bc + c^2 = c^2 + ca + a^2$$

Prove also the converse: if $a^2 + ab + b^2 = b^2 + bc + c^2 = c^2 + ca + a^2$, then $a + b + c = 0$.

Solution Since $(a - c)(a + b + c) = 0$, we have

$$\begin{aligned} a^2 + ab + ac = ac + bc + c^2 &\Rightarrow a^2 + ab = bc + c^2 \\ &\Rightarrow a^2 + ab + b^2 = b^2 + bc + c^2 \end{aligned}$$

Similarly, $b^2 + bc + c^2 = c^2 + ca + a^2$.

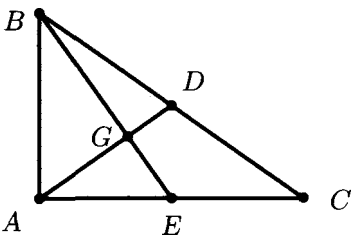
Conversely, we have

$$\begin{aligned} a^2 + ab + b^2 = b^2 + bc + c^2 &\Rightarrow a^2 + ab = bc + c^2 \\ &\Rightarrow (a^2 - c^2) + (ab - bc) = 0 \\ &\Rightarrow (a - c)(a + c) + b(a - c) = 0 \\ &\Rightarrow (a - c)(a + b + c) = 0 \\ &\Rightarrow a + b + c = 0 \text{ since } a \neq c \end{aligned}$$

FINAL – BHASKARA CONTEST
NMTC at JUNIOR LEVEL
IX & X Standards

1. ABC is a right angled triangle with BC as hypotenuse. The medians drawn to BC and AC are perpendicular to each other. If AB has length 1 cm, find the area of the triangle ABC.

Solution Let the medians AD and BE meet at G.



Since ABC is right angled triangle, we have $BD = DC = AD$. Let $AE = x$, $BG = y$ and $GD = z$. Then $AG = 2GD = 2z$, $BD = AD = 3z$, $GE = \frac{1}{2}y$. We have

$$\frac{9}{4}y^2 = BE^2 = AE^2 + AB^2 = x^2 + 1 \qquad (1)$$

$$1 = AB^2 = AG^2 + BG^2 = 4z^2 + y^2 \qquad (2)$$

$$9z^2 = BD^2 = BG^2 + GD^2 = y^2 + z^2 \qquad (3)$$

From (3), we get $y^2 = 8z^2$ and substituting this in (2), we get $z^2 = \frac{1}{12}$ and hence $y^2 = 8z^2 = \frac{2}{3}$. Now, from (1), we get $\frac{9}{4} \times \frac{2}{3} = x^2 + 1$ and hence $x^2 = \frac{1}{2}$. Thus $x = \frac{1}{\sqrt{2}}$

and area of the triangle $ABC = AE \times AB = \dfrac{1}{\sqrt{2}}$ sq cms.

2. (a) Find the smallest positive integer such that it has exactly 100 positive integer divisors including 1 and itself.

   (b) A rectangle can be divided into n equal squares. The same rectangle can also be divided into $n + 76$ equal squares. Find n.

Solution (a) See Primary Final Question 5.

(b) Let the sides of the rectangle be a, b. If x is the side of the square such that n such squares fill the rectangle, then we have $a = k_1 x$ and $b = l_1 x$. Similarly, if y is the side of the square such that $n + 76$ such squares fill the rectangle, we have $a = k_2 y, b = l_2 y$. Note that k_1, l_1, k_2, l_2 are positive integers. Thus

$$\frac{a}{b} = \frac{k_1}{k_2} = \frac{l_1}{l_2} \Rightarrow k_1 l_2 = k_2 l_1$$

Also, $k_1 l_1 = n$ and $k_2 l_2 = n + 76$. Thus

$$n(n + 76) = k_1 l_1 k_2 l_2 = (k_1 l_2)^2 = m^2$$

where m is an integer. Hence

$$\begin{aligned}(n^2 + 76n + 38^2) - 38^2 &= m^2 \\ (n + 38)^2 - m^2 &= 38^2 \\ (n - m + 38)(n + m + 38) &= 38^2 = 1444\end{aligned}$$

The possibilities are

$$\begin{aligned}n - m + 38 &= 1, \quad n + m + 38 = 1444 \\ n - m + 38 &= 2, \quad n + m + 38 = 722 \\ n - m + 38 &= 4, \quad n + m + 38 = 361 \\ n - m + 38 &= 19, \quad n + m + 38 = 76\end{aligned}$$

Of these, only the second set of equations has integer solutions $n = 324$.

Take a 4×1 rectangle. We can divide this into squares of side $\frac{1}{9}$. The number of squares will be 9 rows with 36 squares in each row, and hence a total of $9 \times 36 = 324$ squares. Again we can divide this rectangle into squares of side $\frac{1}{10}$. We will have 10 rows with 40 squares each and hence a total of $400 = 324 + 76$ squares.

3. Prove that $1^n + 2^n + \cdots + 15^n$ is divisible by 480 for all odd integers ≥ 5.

Solution Since $480 = 32 \times 15$, we need to prove that $1^n + 2^n + \cdots + 15^n$ is a multiple of 32 and 15.

Since $n \geq 5$, we have

$$2^n + 4^n + \cdots + 14^n = 2^n(1^n + 2^n + 3^n + \cdots 7^n)$$

is a multiple of 2^n, and hence by 32. We have for any positive integer m,

$$x^{2m+1} + y^{2m+1} = (x+y)(x^{2m} - x^{2m-1}y + \cdots + y^{2m})$$

Now,

$$\begin{aligned}
&1^n+3^n+\cdots 15^n \\
&\quad = (1^n + 15^n) + (3^n + 13^n) + (5^n + 11^n) + (7^n + 9^n) \\
&\quad = 16\{(1^{n-1} - 1^{n-2} \cdot 15 + \cdots + 15^{n-1}) \\
&\qquad + (3^{n-1} - 3^{n-2} \cdot 13 + \cdots + 13^{n-1}) \\
&\qquad + (5^{n-1} - 5^{n-2} \cdot 11 + \cdots + 11^{n-1}) \\
&\qquad + (7^{n-1} - 7^{n-2} \cdot 9 + \cdots + 9^{n-1})\}
\end{aligned}$$

The expression inside the brackets contains an even number of odd terms and hence is a multiple of 2. Thus $1^n + 3^n + \cdots + 15^n$ is a multiple of 32. Since $2^n + 4^n + \cdots + 14^n$ is also a multiple of 32, it follows

that $1^n + 2^n + \cdots + 15^n$ is a multiple of 32.
Clearly 15^n is a multiple of 15. Also,

$$1^n + 2^n + \cdots + 14^n$$
$$= (1^n + 14^n) + (2^n + 13^n) + \cdots + (7^n + 8^n)$$

Each of the terms on the right hand side is a multiple of $1+14$. Thus $1^n+2^n+\cdots+15^n$ is a multiple of 15. This completes the proof.

4. Is it possible to have 19 lines in a plane such that no three lines have a common point and they have exactly 95 points of intersection? Validate.
Solution If no two lines are parallel, then the points of intersection will be $\binom{19}{2} = 171 > 95$. Hence some of the lines must be parallel. If k of the lines are parallel, then the number of intersections is $\binom{19-k}{2} + k(19-k)$. If this equals 95, then we have $k^2 - k - 152 = 0$ and this equation has no integer roots. Hence we must have two sets of parallel lines. Suppose that we have two sets containing k and l parallel lines. The number of points of intersection is

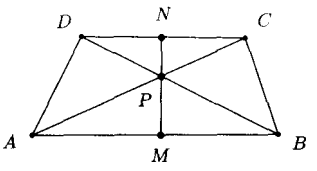
$$171 - \binom{k}{2} - \binom{l}{2} = 95 \Rightarrow k(k-1) + l(l-1) = 152$$

When $k = 11, l = 7$ and when $k = 12, l = 5$ the above equation is satisfied. Thus we can have either 11 lines parallel to each other and another 7 lines parallel to each other and a single line intersecting all the other lines or a set of 12 lines parallel to each other and another 5 lines parallel to each other and two non parallel lines intersecting all the other lines.

5. In a trapezium $ABCD$ with AB parallel to CD, the diagonals intersect at P. The area of ΔABP is 72 cm^2 and of ΔCDP is 50 cm^2. Find the area of the

trapezium.

Solution Let PM, PN be the perpendiculars to the



sides AB and CD respectively. Since $AB \parallel CD$, triangles ABP and CDP are similar and

$$\frac{AB}{CD} = \frac{PM}{PN} = \sqrt{\frac{\frac{1}{2}AB \cdot PM}{\frac{1}{2}CD \cdot PN}} = \sqrt{\frac{72}{50}} = \frac{6}{5}$$

Area of the trapezium $ABCD$ equals

$$\begin{aligned}\frac{1}{2}AB \cdot MN + \frac{1}{2}CD \cdot MN &= \frac{1}{2}AB \cdot (PM + PN) \\ &\quad + \frac{1}{2}CD \cdot (PM + PN)) \\ &= \frac{1}{2}\left(\frac{6}{5}CD \cdot \left(\frac{6}{5}PN + PN\right)\right) \\ &\quad + \frac{1}{2}CD \cdot \left(\frac{6}{5}PN + PN\right) \\ &= \left(\frac{121}{25}\right)\left(\frac{1}{2}CD \cdot PN\right) \\ &= 242\end{aligned}$$

Thus the area of the trapezium is 242 cm^2.

6. Let $a < b < c$ be three positive integers. Prove that among any $2c$ consecutive positive integers, there exists

three different numbers x, y, z such that abc divides xyz.

Solution First, we prove that given positive integers $a < b$, there are two different numbers x, y such that $ab \mid xy$.

Among any b consecutive numbers, there is a number x' such that b divides x': Consider the remainders of the b consecutive numbers when they are divided by b. The remainders are all distinct and must be $0, 1, 2, \ldots, b-1$. Hence there must be one with remainder 0 and that number is divisible by b. Since $a < b$, a similar argument gives that there is a number y' such that $a \mid y'$. If $x' \neq y'$, taking $x = x', y = y'$, our claim is established. If $x' = y'$, then x' is divisible by the least common multiple l of a and b. Let $d = \gcd(a, b)$. Since $a < b$, $d \leq \frac{b}{2}$. Hence there is a number $z' \neq x'$ among the consecutive numbers such that z' is divisible by d. Then $x'z'$ is divisible by $ld = ab$ and we can take $x = x'$ and $y = z'$, establishing the claim in this case.

Suppose now that $a < b < c$. Split the $2c$ numbers into two sets of c consecutive numbers. In the first group, there exist two numbers x, y such that $x \neq y$ and $ab \mid xy$. In the second group, we have a number z such that $c \mid z$. Then abc divides xyz.

7. (a) Let m, n be positive integers. If $m^3 + n^3$ is the square of an integer, then prove that $m + n$ is not a product of two different prime numbers.

   (b) a, b, c are real numbers such that $ab+bc+ca = -1$. Prove that $a^2 + 5b^2 + 8c^2 \geq 4$.

Solution (a) Suppose that $m + n = pq$, where p, q are distinct primes. We can assume that $p \neq 3$. Since $m^3 + n^3 = (m + n)(m^2 - mn + n^2) = pq(p^2q^2 - 3mn)$, and $m^3 + n^3 = k^2$, for some integer k, it follows that pq

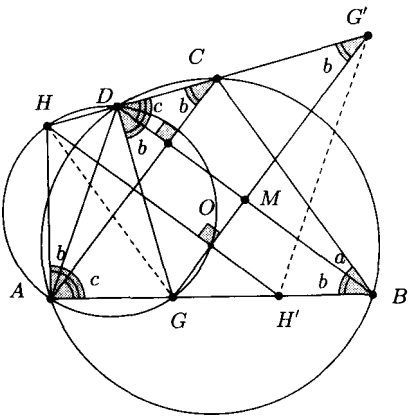
divides k and hence $3mn$. Since $p \neq 3$, either $p \mid m$ or $p \mid n$. Suppose that $p \mid m$. Since $m+n=pq$, it follows that $p \mid n$ also. If $q \mid m$ or $q \mid n$, it follows that it divides the other as well. Thus $pq \mid m$ and $pq \mid n$ giving $m+n \geq 2pq$. Thus $q \nmid m$ and $q \nmid n$. Hence $q=3$. Let $m=kp$ and $n=lp$. Thus $3p=m+n=p(k+l)$. The possibilities are $k=1, l=2$ or $k=2, l=1$. In the first case, $m=p, n=2p$ and $m^3+n^3=9p^3$. Since $p \neq 3$, this is not a perfect square, a contradiction. The other case is similar. Thus $m+n$ is not the product of two distinct prime numbers.

(b)

$$\begin{aligned} a^2+5b^2+8c^2 &= a^2+4b^2+4c^2+b^2+4c^2 \\ &= (a+2b+2cc)^2-4ab-8bc-4ac \\ &\quad +(b-2c)^2+4bc \\ &= (a+2b+2c)^2+(b-2c)^2 \\ &\quad -4(ab+bc+ca) \\ &= (a+2b+2c)^2+(b-2c)^2+4 \\ &\geq 4 \end{aligned}$$

8. $ABCD$ is a quadrilateral inscribed in a circle whose diagonals intersect at right angles. Through O, the center of the circle, GOG' and HOH' are drawn parallel to AC and BD respectively, meeting AB, CD in G, G' and DC, AB in H, H', where the sides of the quadrilateral will be produced if needed. Prove that $GH, G'H'$ are parallel to BC and AD respectively.

Solution Let GG' meet BD at M. Since O is the center of the circle, M is the mid point of BD and $GM \perp BD$. Thus $\angle GBD = \angle GDB = b$ (say). In the cyclic quadrilateral $ABCD$, we have $\angle ABD = \angle ACD = b$. Now, $HH' \perp AC$ and hence $\angle HCA =$



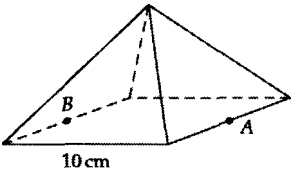
$\angle HAC = b$. Again, from the cyclic quadrilateral $ABCD$, $\angle BDC = \angle BAC = c$ (say). Now, $\angle HAC = b + c$ and $GDC = b + c$. Thus the quadrilateral $AHDG$ is cyclic. Hence $\angle HGA = \angle HDA = \angle ABC$ (using the cyclic quadrilateral $ABCD$). Thus $HG \parallel BC$.

The proof that $H'G' \parallel AD$ is similar (observe that $HGH'G'$ is also a cyclic quadrilateral).

FINAL – RAMANUJAN CONTEST
NMTC at INTER LEVEL
XI & XII Standards

1. A pyramid made from a solid block of wood rests with its square base on a flat table top. Each edge of the square base has length 10 cm. The lateral sides of the pyramid are equilateral triangles. An ant is walking on the sides of the pyramid from the midpoint A of one of the edges of the base to the midpoint B of the opposite edge. See the diagram. Find the length of the shortest path the ant has to walk.

Solution Suppose we flatten the pyramid. If we refer to



the Figure 1, which shows three of the four faces of the flattened pyramid, then we see that the shortest distance from point A to point B is a straight line through the midpoints of the three triangles. Since the base of the

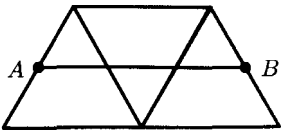


Figure 1

triangle has length, the line joining a pair of mid points of sides has length 5 and thus the shortest distance between A and B is 15 cm.

2. Six children stand in a line outside their classroom. They enter the classroom and sit in a circular table. In how many ways can they sit such that no two children who were standing next to each other outside are sitting next to each other at the table?

Solution Suppose that the order in which they stand outside their classroom is denoted by $1, 2, 3, 4, 5, 6$. We need to find the number of ways in which the integers $1, 2, \ldots, 6$ can be placed around a circle such that no two adjacent integers are consecutive. Let us first place the integers $1, 2, 3$ on the circle. We can place these in two orientations, as shown in Figure 2. We need to place the

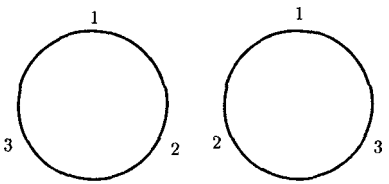


Figure 2

remaining $4, 5, 6$ in the gaps. Note that there must be at least one child between 1, 2 and between 2, 3. There are three cases to consider:

(a) One child between 1 and 2 and one child between 2 and 3
(b) Two children between 1 and 2 and one child between 2 and 3
(c) One child between 1 and 2 and two children between 2 and 3.

In (a), child 4 can not be placed between 2 and 3 or between 3 and 1. Hence she must be placed between 1 and 2. Once child 4 is placed, 5 and 6 can be placed in any of the other two places. Hence there are two admissible

arrangements in this case.

In (b), the two children placed between 1 and 2 must be $4, 6$. Also we can place them in two ways. Hence there are 2 ways to place the children in this case.

In (c), if we place child 5 between 1 and 2, we can place 4 and 6 between 2 and 3 in one way. We can not place child 6 or 4 between 1 and 2, since we will be left with either $4, 5$ or $5, 6$ and both are adjacent. Hence we have another 1 arrangement in this case. Thus we have a total of 5 arrangements when we place $1, 2, 3$ in the clockwise order around the circle. We have another 5 ways of placing the children corresponding to the anticlockwise placement of $1, 2, 3$. Thus there are 10 ways in which no two children who were standing adjacent are next to each other in the circle.

We have tacitly assumed that the placements obtained by rotating an arrangement around the circle is not considered as a different arrangement.

3. A 5×5 square is divided into 25 unit squares by lines drawn parallel to its sides. Some of these unit squares are colored red and others colored blue. Show that we can always find 4 squares that are of the same color and such that they are the corners of a rectangle with sides parallel to the square.

Solution 1 We will show that if 13 or more squares have the same color, then there is a rectangle whose corners have that color. By Pigeonhole Principle, there are at least 13 squares with the same color, say red. Hence it is sufficient to prove the above assertion.

Two squares that lie in the same column and have the

same color will be called a *doublet*. Let $a_1, a_2, \ldots, a_5$ be the number of red squares in the columns $1, 2, \ldots, 5$ respectively. Then $\sum a_i = 13$. Also,

$$\begin{aligned}\sum_{i=1}^{5} \binom{a_i}{2} &= \frac{1}{2}\left(\sum_{i=1}^{5} a_i^2 - 13\right) \\ &\geq \frac{1}{2}\left(5\left(\frac{1}{5}\sum a_i\right)^2 - 13\right) \\ &= \frac{104}{10}\end{aligned}$$

Note that we have used the generalized mean inequality,

$$\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right)^2 \leq \frac{1}{n}\sum_{i=1}^{n} a_i^2$$

Thus

$$\sum_{i=1}^{5} \binom{a_i}{2} \geq 11$$

Since there are only $\binom{5}{2} = 10$ pairs of rows, two of the doublets in the columns determine the same set of two rows. Thus there is a rectangle all of whose corners are red.

Solution 2 As observed in Solution 1, there are at least 13 squares that have the same color.

Case 1: All squares in a column are red.

There are 8 more squares of that color and we have 4 columns. Hence at least one must contain two squares with the same color. Those two squares along with the squares in their rows in the all red column will form the desired rectangle.

Case 2: There is a column with 4 red squares.

We have 9 more red squares distributed among 4 other columns and hence there is at least one that has 3 red squares. Thus the number of common rows that have red squares between this column and the 4 red square column is at least 2 and hence there is a red rectangle in these two columns.

Case 3: No column has more than 3 red squares.

The number of columns with 3 red squares is at least 3: If there are only 2 columns with 3 squares, the other 3 columns must have 7 red squares and hence not all can have only 2 red squares). Suppose that the set of rows of the red squares in these columns be denoted by R_1, R_2, R_3. If we prove at least one of $|R_i \cap R_j| \geq 2$, we are through. Suppose on the contrary that $|R_i \cap R_j| = 1$ for all i, j (note that $R_i \cap R_j \neq \phi$). Now, by Principle of Inclusion Exclusion,

$$\begin{aligned} |R_1 \cup R_2 \cup R_3| &= |R_1| + |R_2| + |R_3| \\ &\quad - |R_1 \cap R_2| - |R_2 \cap R_3| - |R_3 \cap R_1| \\ &\quad + |R_1 \cap R_2 \cap R_3| \\ &= 9 - 3 + |R_1 \cap R_2 \cap R_3| \\ &\geq 6 \end{aligned}$$

a contradiction. Thus some two columns have two common rows and there is a red rectangle.

4. $a, b, c > 0$ and $a^2 + b^2 + c^2 = 1$. Show that

$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} \geq 3 + \frac{2(a^3 + b^3 + c^3)}{abc}$$

Solution We need

$$(a^2 + b^2 + c^2)\left(\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}\right) \geq 3 + \frac{2(a^3 + b^3 + c^3)}{abc}$$

Expanding the left hand side, this is equivalent to

$$\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}+\frac{a^2}{c^2}+\frac{b^2}{a^2}+\frac{c^2}{b^2}\geq 2\left(\frac{a^2}{bc}+\frac{b^2}{ca}+\frac{c^2}{ab}\right)$$

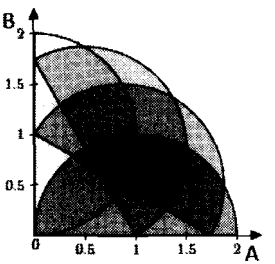
Now, for any two real numbers x, y, $x^2+y^2 \geq 2xy$. Hence

$$\begin{aligned}\frac{a^2}{b^2}+\frac{a^2}{c^2} &\geq 2\frac{a^2}{bc}\\ \frac{b^2}{c^2}+\frac{b^2}{a^2} &\geq 2\frac{b^2}{ac}\\ \frac{c^2}{a^2}+\frac{c^2}{b^2} &\geq 2\frac{c^2}{ab}\end{aligned}$$

Adding we obtain the desired inequality.

5. O, A, B are points such that $OA \perp OB$, $OA = OB = 2$. For any point P on OA and Q on OB such that $PQ = 2$, semi circles are drawn on PQ as diameter such that all those semicircles lie between lines OA, OB. Find the total area covered by these semi circles.

Solution From the picture above, we see that the union



of the half-disks will be a quarter-circle with radius 2, and therefore area π. To prove that this is the case, we first prove that the boundary of every half-disk intersects the quarter-circle with radius 2, and then that the half-disk is

internally tangent to the quarter-circle at that point. This is sufficient because it is clear from the diagram that we need not worry about covering the interior of the quarter-circle.

Consider any half disk. Draw perpendiculars at D and

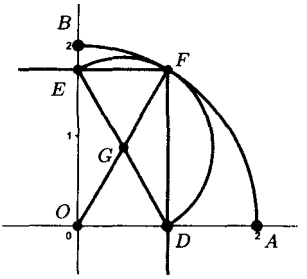


Figure 3

E to meet at F. Then $OF = DE = 2$ and hence F lies on the quarter circle. Also, the center of this half disk is G, the mid point of DE and the distance between this center and the center O of the quarter circle is equal to $2 - 1$, the difference between their radii and hence the half disk touches the quarter circle and we are through.

6. Find the largest possible value of n such that 3^{11} can be expressed as a sum of n consecutive positive integers.

Solution Suppose that

$$3^{11} = (k+1) + \cdots + (k+n) = kn + \frac{n(n+1)}{2}$$

Hence $2 \cdot 3^{11} = n(2k+n+1)$. Thus n divides $2 \cdot 3^{11}$ and hence must be of the form $2^i \cdot 3^j$, where $i = 0$ or 1 and

$0 \leq j \leq 11$. There are only 24 possibilities for (i, j) and hence there are only 24 possible values for k. We check these out and eliminate the impossibilities.

Let $i = 1$. Then

$$2 \cdot 3^{11} = 2 \cdot 3^j(2k + 2 \cdot 3^j + 1)$$
$$3^{11-j} = 2k + 2 \cdot 3^j + 1$$

If $11 - j < j$, the right hand side will exceed the left hand side. Thus $11 - j > j$ and hence $j \leq 5$. If $j = 5$, we have

$$3^6 = 2k + 2 \cdot 3^5 + 1 \Rightarrow k = (3^5 - 1)/2 = 40$$

and $n = 2 \cdot 243 = 486$ works.

If $i = 0$, then

$$2 \cdot 3^{11} = 3^j(2k + 3^j + 1)\ 2 \cdot 3^{11-j} = 2k + 3^j + 1$$

If $j \geq 6$, right hand side exceeds the left hand side. Thus $j \leq 5$. But the value of n obtained will be less than 486 and hence we need not consider these values. Hence the maximum n is 486.

7. The bisector of angle BAD in the parallelogram $ABCD$ intersects the lines BC and CD at K and L respectively. It is given that $ABCD$ is not a rhombus. Prove that the center of the circle passing through the points C, K and L lies on the circle passing through the points B, C and D.

Solution Let $\angle BAD = 2x$. Since BL bisects $\angle BAD$, $\angle CKL = \angle CLK = x$ and hence $KC + CL$. If O is the center of the circle through K, C, L, then $OC \perp KL$. $\angle KOC = 2x$ and $\angle OKC = 90° - 2x$. Thus $\angle BKO =$

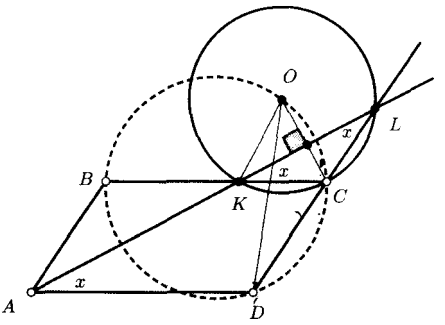


Figure 4

$90° + x = \angle DCO$.
But $BK = AB = DC$ and $OK = OC$. Thus triangles BKO and DCO are congruent. In particular, $\angle BOK = \angle DOC$. Thus $\angle BOD = \angle KOC$. Now,

$$\angle KOC = 2\angle KLC = 2x = \angle BCD$$

and hence $\angle BOD = \angle BCD$, showing that the points B, O, C and D lie on a circle.

8. Three unit circles C_1, C_2 and C_3 in the plane have the property that each circle passes through the centers of the other two. A square S surrounds the three circles in such a way that each of its four sides is tangent to at least one of C_1, C_2 and C_3. Find the length of the side of the square S.

Solution If we are carried away by symmetry, we will draw the wrong figure 5: As we can see here, we do not get a square. The correct way is as in Figure 6.

By the Pigeonhole Principle, two of the sides must be

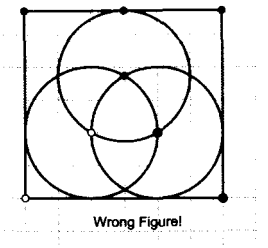


Figure 5

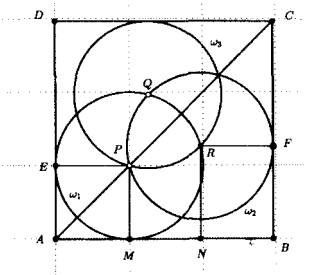


Figure 6

tangent to the same circle, say ω_1. Since S surrounds the circles, these two sides must be adjacent, so we can let A denote the common vertex of the two sides tangent to ω_1. Let B, C, and D be the other vertices of S in clockwise order, and let P, Q, and R be the centers of ω_1, ω_2 and ω_3 respectively, and suppose WLOG that they are also in clockwise order. Then AC passes through the center of ω_1, and by symmetry (since $AB = AD$) it must also pass through the other intersection point of ω_2 and ω_3. That is, AC is the radical axis of ω_2 and ω_3.

Now, let M and N be the feet of the perpendiculars from P and R, respectively, to side AD. Let E and F be the feet of the perpendiculars from P to AB and from

R to DC, respectively. Then $PEAM$ and $NRFD$ are rectangles, and PE and RF are radii of ω_1 and ω_2 respectively. Thus $AM = EP = 1$ and $ND = RF = 1$.

Finally,

$$\begin{aligned} MN &= PR\cos(180^\circ - \angle EPR) \\ &= \cos(180^\circ - \angle EPQ - \angle RPQ) \\ &= -\cos(\angle EPQ + \angle RPQ) \end{aligned}$$

$\angle RPQ = 60^\circ$ and $\angle EPQ = (270^\circ - 60^\circ)/2$, since AP bisects $\angle EPM$. Thus $MN = -\cos 165^\circ = \cos 15^\circ = \dfrac{\sqrt{6}+\sqrt{2}}{4}$.

Side of the square is

$$AD + MN + NB = 1 + \frac{\sqrt{6}+\sqrt{2}}{4} + 1 = \frac{\sqrt{6}+\sqrt{2}+8}{4}$$

Pre RMO 2018

1. A book is published in three volumes, the pages being numbered from 1 onwards. The page numbers are continued from the first volume to the second volume to the third. The number of pages in the second volume is 50 more than that in the first volume, and the number pages in the third volume is one and a half times that in the second. The sum of the page numbers on the first pages of the three volumes is 1709. If n is the last page number, what is the largest prime factor of n?

2. In a quadrilateral $ABCD$, it is given that $AB = AD = 13$, $BC = CD = 20$, $BD = 24$. If r is the radius of the circle inscribable in the quadrilateral, then what is the integer closest to r?

3. Consider all 6-digit numbers of the form $abccba$ where b is odd. Determine the number of all such 6-digit numbers that are divisible by 7.

4. The equation $166 \times 56 = 8590$ is valid in some base $b \geq 10$ (that is, $1, 6, 5, 8, 9, 0$ are digits in base b in the above equation). Find the sum of all possible values of $b \geq 10$ satisfying the equation.

5. Let $ABCD$ be a trapezium in which $AB \parallel CD$ and $AD \perp AB$. Suppose $ABCD$ has an incircle which touches AB at Q and CD at P. Given that $PC = 36$ and $QB = 49$, find PQ.

6. Integers a, b, c satisfy $a+b-c = 1$ and $a^2+b^2-c^2 = -1$. What is the sum of all possible values of $a^2 + b^2 + c^2$?

7. A point P in the interior of a regular hexagon is at distances $8, 8, 16$ units from three consecutive vertices of the hexagon, respectively. If r is radius of the

circumscribed circle of the hexagon, what is the integer closest to r?

8. Let AB be a chord of a circle with centre O. Let C be a point on the circle such that $\angle ABC = 30^\circ$ and O lies inside the triangle ABC. Let D be a point on AB such that $\angle DCO = \angle OCB = 20^\circ$. Find the measure of $\angle CDO$ in degrees.

9. Suppose a, b are integers and $a+b$ is a root of $x^2+ax+b=0$. What is the maximum possible value of b^2?

10. In a triangle ABC, the median from B to CA is perpendicular to the median from C to AB. If the median from A to BC is 30, determine $(BC^2+CA^2+AB^2)/100$.

11. There are several tea cups in the kitchen, some with handles and the others without handles. The number of ways of selecting two cups without a handle and three with a handle is exactly 1200. What is the maximum possible number of cups in the kitchen?

12. Determine the number of 8-tuples $(\epsilon_1, \epsilon_2, \ldots, \epsilon_8)$ such that $\epsilon_1, \epsilon_2, \ldots, \epsilon_8 \in \{1, -1\}$ and

$$\epsilon_1 + 2\epsilon_2 + 3\epsilon_3 + \cdots + 8\epsilon_8$$

is a multiple of 3.

13. In a triangle ABC, right-angled at A, the altitude through A and the internal bisector of $\angle A$ have lengths 3 and 4, respectively. Find the length of the median through A.

14. If $x = \cos 1^\circ \cos 2^\circ \cos 3^\circ \cdots \cos 89^\circ$ and $y = \cos 2^\circ \cos 6^\circ \cos 10^\circ \cdots \cos 86^\circ$, then what is the integer nearest to $\frac{2}{7}\log_2(y/x)$?

15. Let a and b be natural numbers such that $2a-b, a-2b$ and $a+b$ are all distinct squares. What is the smallest possible value of b?

16. What is the value of

$$\sum_{\substack{1\le i<j\le 10 \\ i+j=\text{ odd}}} (i+j) - \sum_{\substack{1\le i<j\le 10 \\ i+j=\text{ even}}} (i+j)$$

17. Triangles ABC and DEF are such that $\angle A = \angle D$, $AB = DE = 17, BC = EF = 10$ and $AC - DF = 12$. What is $AC + DF$?

18. If $a,b,c \ge 4$ are integers, not all equal, and $4abc = (a+3)(b+3)(c+3)$, then what is the value of $a+b+c$?

19. Let $N = 6 + 66 + 666 + \cdots + 666\cdots 66$, where there are hundred 6's in the last term in the sum. How many times does the digit 7 occur in the number N?

20. Determine the sum of all possible positive integers n, the product of whose digits equals $n^2 - 15n - 27$.

21. Let ABC be an acute-angled triangle and let H be its orthocentre. Let G_1, G_2 and G_3 be the centroids of the triangles HBC, HCA and HAB, respectively. If the area of triangle $G_1G_2G_3$ is 7 units, what is the area of triangle ABC?

22. A positive integer k is said to be good if there exists a partition of $\{1,2,3,\cdots,20\}$ in to disjoint proper subsets such that the sum of the numbers in each subset of the partition is k. How many good numbers are there?
Solution If there are n sets in the partition, then we need $nk = \sum_{i=1}^{20} i = 210$. Thus k must be a divisor of $210 = 2 \cdot 3 \cdot 5 \cdot 7$. Note that $k \ge 20$, since the number 20 must be present in one of the sets of the

partition and since k must also divide 210, it follows that $k > 20$. The proper divisors that are more than 20 are $21, 30, 35, 42, 70, 105$. We will show that each of these values is possible.

(1) $k = 21$: The partition is

$$\{1, 20\}, \{2, 19\}, \ldots, \{10, 11\}$$

(2) $k = 30$: We can partition as follows:

$$\{10, 20\}, \{11, 19\}, \{12, 18\}, \{13, 17\}, \{14, 16\},$$
$$\{6, 9, 15\}, \{1, 2, 3, 4, 5, 7, 8\}$$

(3) $k = 35$:

$$\{5, 9, 10, 11\}, \{6, 7, 8, 14\}, \{4, 15, 16\}, \{17, 18\},$$
$$\{2, 13, 20\}, \{1, 3, 12, 19\}$$

(4) $k = 42$: Join pairs of sets from the partition in (1).
(5) $k = 70$: Join pairs of sets from the partition in (3).
(6) $k = 105$: Join three sets from the partition in (3).

Thus there are six such numbers.

23. What is the largest positive integer n such that

$$\frac{a^2}{\frac{b}{29} + \frac{c}{31}} + \frac{b^2}{\frac{c}{29} + \frac{a}{31}} + \frac{c^2}{\frac{a}{29} + \frac{b}{31}} \geq n(a + b + c)$$

holds for all positive real numbers a, b, c.

Solution It is not difficult to show that the following inequality holds:

$$\frac{a^2}{x} + \frac{b^2}{y} + \frac{c^2}{z} \geq \frac{(a + b + c)^2}{x + y + z}$$

Hence

$$\begin{aligned}
&\frac{a^2}{\frac{b}{29}+\frac{c}{31}}+\frac{b^2}{\frac{c}{29}+\frac{a}{31}}+\frac{c^2}{\frac{a}{29}+\frac{b}{31}} \\
&\qquad \geq \frac{(a+b+c)^2}{\left(\frac{1}{29}+\frac{1}{31}\right)(a+b+c)} \\
&\qquad = \frac{29\cdot 31}{60}(a+b+c) \\
&\qquad \geq 14(a+b+c)
\end{aligned}$$

When $a=b=c=1$, the left hand side evaluates to $44\frac{19}{20}$ and $15(a+b+c)=45$. Thus the largest n is 14.

24. If N is the number of triangles of different shapes (i.e., not similar) whose angles are all integers (in degrees), what is $N/100$?

Solution Let x, y, z be the angles of the triangle. Then $x+y+z=180°$. We can assume that $x \geq y \geq z$. We have one triangle when $x=y=z$. When $x=y>z$, the possible triangles have angles $(61,61,58),(62,62,56),\ldots,(89,89,2)$ and when $x>y=z$, the possible triangles are $(178,1,1),(176,2,2),\ldots,(62,59,59)$. These give a total of 88 triangles.

The number of positive integer solutions to the equation $x+y+z=180°$ is $\binom{179}{2}=15931$. The triangles in which $x=y$ or $y=z$ are counted three times in the list of solutions to this equation. The equilateral triangle is counted once and the rest are counted 3! times. The unique triangles with $x>y>z$ is therefore $\frac{15931-3\times 88-1}{3!}=2611$. Hence the number of unique triangles is $N=2611+88+1=2700$ and $N/100=27$.

25. Let T be the smallest positive integer which, when divided by $11, 13, 15$ leaves remainders in the sets $\{7,8,9\}, \{1,2,3\}, \{4,5,6\}$ respectively. What is the sum of the squares of the digits of T?
Solution The number T is of the form $15n+4$ or $15n+5$ or $15n+6$. Suppose that T is of the form $15n+4$. Then

$$15n + 4 = (2n + 4) \pmod{13},$$
$$15n + 4 = (4n + 4) \pmod{11}$$

We need $2n + 4 = 1 \pmod{13}$ or $2 \pmod{13}$ or $3 \pmod{13}$ and $4n+4 = 7 \pmod{11}$ or $8 \pmod{11}$ or $9 \pmod{13}$. The possible values of n

26. What is the number of ways in which one can choose 60 unit squares from a 11×11 chessboard such that no two chosen squares have a side in common?

27. What is the number of ways in which one can color the squares of a 4×4 chessboard with colours red and blue such that each row as well as each column has exactly two red squares and two blue squares?
Solution 1 Any row can be colored in six ways $RRBB, RBRB, RBBR, BBRR, BRBR, BRRB$. We show that for any choice of first row, there are 15 colorings. Hence the number of possible colorings is $6 \times 15 = 90$.

Suppose that the first row is $RRBB$. Consider the columns under the ones colored R. These can be one of the possible three: RBB, BRB, BBR. Consider the case

R	R	B	B
R			
B			
B			

The other column under R also has three possibilities: RBB, BRB, BBR. If it is RBB, there is only one possibility to color the other squares. If it is BRB, the last row has only one possibility and the second row has two possibilities - RB or BR. Since this fixes the third row as well, it follows that in this case we have 2 possibilities. Similarly, when the second column is BBR, we have two possibilities. Thus for this choice of first column and first row, there are 5 ways to color the board. Similar argument shows that there are 5 possibilities when the first column is BRB or BBR. Thus we have 15 possibilities when we choose a first row. Since there are six choices for the first row, we get 90 possibilities.

Solution 2 Any row can be colored in six ways. The first two rows can be colored in $6 \times 6 = 36$ ways. We split the cases according to the number of cells in the first two rows that are identical.

Case 1 There are six colorings in which the first two rows are identical. There is only one way to color the other two rows.

Case 2 First row and second row differ at all columns. Here there are six ways to color the first row and there is one way to color the second row such that it differs from the first row at all columns. Third row can be colored in six ways and this fixes the colors for the fourth row. Thus there are $6 \times 6 = 36$ colorings.

Case 3 First two rows differ at two columns. There are $36 - 6 - 6 = 24$ such colorings. For each such coloring, when the colors are the same in the first two rows, such a column can be colored in only one way. The cells in the

third row corresponding to the two columns that differ in the first two rows can be colored in two ways. This fixes the fourth row color as well. Thus there are $2 \times 24 = 48$ such colorings.

Note that first two rows can not differ in only one column. Thus the total number of colorings is $6 + 36 + 48 = 90$.

28. Let N be the number of ways of distributing 8 chocolates of different brands among 3 children such that each child gets at least one chocolate, and no two children get the same number of chocolates. Find the sum of the digits of N.

Solution The distinct partitions of 8 are $1 + 2 + 5$ and $1 + 3 + 4$. The number of ways of distributing the chocolates is

$$3!\left(\binom{8}{1}\binom{7}{2} + \binom{8}{1}\binom{7}{3}\right) = 2688$$

Thus the sum of the digits is $2 + 6 + 8 + 8 = 24$.

29. Let D be an interior point of the side BC of a triangle ABC. Let I_1 and I_2 be the incentres of triangles ABD and ACD respectively. Let AI_1 and AI_2 meet BC in E and F respectively. If $\angle BI_1E = 60°$, what is the measure of $\angle CI_2F$ in degrees?

30. Let $P(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ be a polynomial in which a_i is a non-negative integer for each $i \in \{0, 1, 2, 3, \ldots, n\}$. If $P(1) = 4$ and $P(5) = 136$, what is the value of $P(3)$?

Solution Since $P(1) = 4$, we have $a_0 + a_1 + \cdots + a_n = 4$. Also, $136 = 5^3 + 2 \cdot 5 + 1$ and hence if $a_0 = 1, a_1 = 2$ and $a_3 = 1$, then we have both conditions satisfied. Thus $P(x) = 1 + 2x + x^3$ and $P(3) = 34$.

RMO 2018

1. Let ABC be a triangle with integer sides in which $AB < AC$. Let the tangent to the circumcircle of triangle ABC at A intersect the line BC at D. Suppose AD is also an integer. Prove that $\gcd(AB, AC) > 1$.

   Solution We may assume that B lies between C and D. Let $AB = c, BC = a$ and $CA = b$. Then $b > c$. Let $BD = x$ and $AD = y$. Observe that $\angle DAB = \angle DCA$. Hence $\Delta DAB \sim \Delta DCA$. We get $\frac{x}{y} = \frac{c}{b} = \frac{y}{x+a}$. Therefore $xb = yc$ and $by = c(x + a)$. Eliminating x, we get $y = abc/(b^2 - c^2)$. Suppose $\gcd(b, c) = 1$. Then $\gcd(b, b^2 - c^2) = 1 = \gcd(c, b^2 - c^2)$. Since y is an integer, $b^2 - c^2$ divides a. Therefore $b + c$ divides a. Hence $a < b + c$. This contradicts triangle inequality. We conclude that $\gcd(b, c) > 1$.

2. Let n be a natural number. Find all real numbers x satisfying the equation

$$\sum_{k=1}^{n} \frac{kx^k}{1 + x^{2k}} = \frac{n(n+1)}{4}$$

   Solution Observe that $x \neq 0$. We have

$$\begin{aligned} \frac{n(n+1)}{4} &= \left| \sum_{k=1}^{n} \frac{kx^k}{1 + x^{2k}} \right| \\ &\leq \sum_{k=1}^{n} \frac{k|x|^k}{1 + x^{2k}} = \sum_{k=1}^{n} \frac{k}{1/|x|^k + |x|^k} \\ &\leq \sum_{k=1}^{n} \frac{k}{2} = \frac{n(n+1)}{4} \end{aligned}$$

   Hence equality holds everywhere and $x = |x|$ and $|x| = 1/|x|$. Thus $x = 1$ is the only possibility.

3. For a rational number r, its period is the length of the smallest repeating block in its decimal expansion. For example, the number $r = 0.123123123\ldots$ has period 3. If S denotes the set of all rational numbers r of the form $r = 0.abcdefgh$ having period 8, find the sum of all the elements of S.

   Solution Let us first count the number of elements in S. There are 10^8 ways of choosing a block of length 8. Of these, we should not count the blocks of the form $abcdabcd, abababab$, and $aaaaaaaa$. There are 10^4 blocks of the form $abcdabcd$. They include blocks of the form $abababab$ and $aaaaaaaa$. Hence the blocks of length exactly 8 is $10^8 - 10^4 = 99990000$.

   Now, for each block $abcdefgh$ consider the block $a'b'c'd'e'f'g'h'$ where $x' = 9 - x$. Observe that whenever $0.\overline{abcdefgh}$ is in S, the rational number $0.\overline{a'b'c'd'e'f'g'h'}$ is also in S. Thus every element $0.\overline{abcdefgh}$ of S can be uniquely paired with a distinct element $0.\overline{a'b'c'd'e'f'g'h'}$ of S. We also observe that

$$0.\overline{abcdefgh} + 0.\overline{a'b'c'd'e'f'g'h'} = 0.\overline{99999999} = 1$$

   Hence the sum of elements in S is $\dfrac{99990000}{2} = 49995000$.

4. Let E denote the set of 25 points (m, n) in the xy-plane, where m, n are natural numbers, $1 \leq m \leq 5$, $1 \leq n \leq 5$. Suppose the points of E are arbitrarily colored using two colors, red and blue. Show that there always exist four points in the set E of the form $(a, b), (a + k, b), (a + k, b + k), (a, b + k)$ for some positive integer k such that at least three of these four points have the same color. (That is, there always exist four points in the set E which form the vertices of a square

and having at least three points of the same color.)

Solution Name the points from bottom row to top (and from left to right) as A_j, B_j, C_j, D_j, E_j , $1 \leq j \leq 5$. Note that among 5 points A_1, B_1, C_1, D_1, E_1 , there are at least 3 points of the same color, say, red. (This follows from pigeonhole principle.) We consider several cases: (the argument holds irrespective of the color assigned to the other two points.)

(a) Take three adjacent points having the same color. (e.g. A_1, B_1, C_1 or B_1, C_1, D_1 .) The argument is similar in both the cases. If A_1, B_1, C_1 are red then A_2, B_2, C_2 are all blue; otherwise we get a square having three red vertices. The same reasoning shows that A_3, B_3, C_3 are all red. Now $A_1, C1, A_3, C_3$ are red vertices and form a square.

(b) Three alternate points A_1, C_1, E_1 are red: Then A_3, C_3, E_3 have to be blue; otherwise, we get a square with three red vertices. Same reasoning shows that A_5, C_5, E_5 are red. Therefore we have A_1, E_1, A_5, E_5 have red color.

(c) Only two adjacent points having red color: There are three sub cases.
(a) A_1, B_1, D_1 red: In this case A_2, B_2 are blue and therefore A_3, B_3 are red. But then B_1, D_1, B_3 are vertices of a square and all are red.
(b) B_1, C_1, E_1 are red. This is similar to case (a)
(c) A_1, B_1, E_1 are red. We successively have A_2, B_2 blue, A_3, B_3 red, $A - 4, B_4$ blue and A_5, B_5 red. Now, A_1, E_1, A_5 are red vertices of a square.

These are the only essential cases and all other cases reduce to one of these.

5. Find all natural numbers n such that $1 + \lfloor \sqrt{2n} \rfloor$ divides

$2n$. (For any real number x, $\lfloor x \rfloor$ denotes the largest integer not exceeding x.)

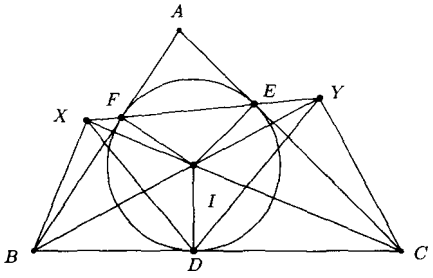
Solution Let $\lfloor \sqrt{2n} \rfloor = k$. We observe that for any real number x, $x - 1 < \lfloor x \rfloor \leq x$. Hence $\sqrt{2n} < 1 + k \leq 1 + \sqrt{2n}$. Divisibility gives $(1+k)d = 2n$ for some positive integer d. Hence $\sqrt{2n} < \dfrac{2n}{d} \leq 1 + \sqrt{2n}$. From the first inequality, $d < \sqrt{2n} < 1 + k$. Hence

$$d = \frac{2n}{1+k} = \frac{(\sqrt{2n})^2}{1+k} \geq \frac{k^2}{1+k} = (k-1) + \frac{1}{k+1} > k - 1$$

We thus obtain $k - 1 < d < k + 1$. Since d is an integer, $d = k$. This implies that $n = \dfrac{k(k+1)}{2}$. Thus n is a triangular number. It is easy to check that any triangular number is a solution.

6. Let ABC be an acute-angled triangle with $AB < AC$. Let I be the incentre of triangle ABC, and let D, E, F be the points at which its incircle touches the sides BC, CA, AB, respectively. Let BI, CI meet the line EF at Y, X, respectively. Further assume that both X and Y are outside the triangle ABC. Prove that
(a) B, C, Y, X are concyclic and
(b) I is also the incentre of triangle DYX.

Solution (a) We first show that $BIFX$ is a cyclic quadrilateral. Since $\angle BIC = 90^\circ + (A/2)$, we see that $\angle BIX = 90^\circ - (A/2)$. On the other hand FAE is an isosceles triangle so that $\angle AFE = 90^\circ - (A/2)$. But $\angle AFE = \angle BFX$ as they are vertically opposite angles. Therefore $\angle BFX = 90^\circ - (A/2) = \angle BIX$. It follows that $BIFX$ is a cyclic quadrilateral. Therefore $\angle BXI = \angle BFI$. But $\angle BFI = 90^\circ$ since $IF \perp AB$. We obtain $\angle BXC = \angle BXI = 90^\circ$.
A similar consideration shows that $\angle BYC = 90^\circ$.



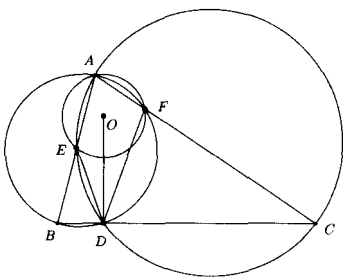
Therefore $\angle BXC = \angle BYC$ which implies that $BCYX$ is a cyclic quadrilateral.

(b) We also observe that $BDIX$ is a cyclic quadrilateral as $\angle BXI = 90^\circ = \angle BDI$ and therefore $\angle BXI + \angle BDI = 180^\circ$. This gives $\angle DXI = \angle DBI = (B/2)$. Since B, I, F, X are concyclic, $\angle IXF = \angle IBF = B/2$. Hence $\angle DXI = \angle IXF$. Hence XI bisects $\angle DXY$. Similarly, we can show that YI bisects $\angle DYX$. It follows that I is the incentre of ΔDYX as well.

RMO 2018
(Tamilnadu and Kerala Regions)

1. Let ABC be an acute-angled triangle and let D be an interior point of the line segment BC. Let the circumcircle of triangle ACD intersect AB at E (E between A and B) and let the circumcircle of triangle ABD intersect AC at F (F between A and C). Let O be the circumcentre of triangle AEF. Prove that OD bisects $\angle EDF$.

Solution Observe that $ACDE$ is a cyclic quadrilateral.



Hence $\angle BDE = \angle EAC = \angle A$. Similarly, from the cyclic quadrilateral $ABDF$, we get $\angle CDF = \angle FAB = \angle A$. We also have

$$\angle EDF = 180^\circ - \angle BDE - \angle CDF = 180^\circ - 2\angle A$$

Since $\angle A$ is acute, O and D lie on opposite sides of EF. Hence $\angle EOF = 2\angle A$. Thus $\angle EDF + \angle EOF = 180^\circ$ and $EOFD$ is a cyclic quadrilateral. Since $OE = OF$, we have $\angle OEF = \angle OFE$. Finally, cyclicity of $EOFD$ gives $\angle ODE = \angle OFE = \angle OEF = \angle ODF$. Thus OD bisects $\angle EDF$.

2. Find the set of all real values of a for which the real polynomial equation $P(x) = x^2 - 2ax + b = 0$ has real roots given that $P(0) \cdot P(1) \cdot P(2) \neq 0$ and $P(0), P(1), P(2)$ form a geometric progression.

   Solution Since none of $P(0), P(1), P(2)$ are zero and they form a geometric progression, we have $P(1)^2 = P(0)P(2)$ giving $(1 - 2a + b)^2 = (4 - 4a + b)b$. This simplifies to $(2a - 1)^2 = 2b$. Now, $P(x) = 0$ has real roots if and only if $4a^2 \geq 4b$. Hence $2a^2 \geq 2b = (2a - 1)^2$ and thus $2a^2 - 2a + 1 \leq 0$. Since the roots of $2x^2 - 2x + 1 = 0$ are $\frac{2 \pm \sqrt{2}}{2}$, a lies in the interval $\left[\frac{2 - \sqrt{2}}{2}, \frac{2 + \sqrt{2}}{2}\right]$

3. Show that there are infinitely many 4-tuples (a, b, c, d) of natural numbers such that $a^3 + b^4 + c^5 = d^7$.

   Solution Observe that the least common multiple of $3, 4, 5$ is 60. First we seek a solution with $a^3 = b^4 = c^5 = l^{60k}$ where we choose l, k such that the condition is satisfied. The given equation gives $3l^{60k} = d^7$. This suggests that we need to choose $l = 3$ and $60k + 1$ is divisible by 7. When $k = 5$, we have $60k + 1 = 301 = 7 \times 43$, is a multiple of 7. Thus we have $3^{100} + 3^{100} + 3^{100} = 3^{101}$ and we can take $a = 3^{100}, b = 3^{75}, c = 3^{60}$ and $d = 3^{43}$ to obtain a solution.
   Now, we can take $a = 3^{100} \cdot m^{140}, b = 3^{75} \cdot m^{105}, c = c^{60} \cdot m^{84}$ and $d = 3^{43} \cdot m^{60}$. Then

   $$a^3 + b^4 + c^5 = 3^{301} \cdot m^{420} = \left(3^{43} \cdot m^{60}\right)^7$$

   Since the above holds for an arbitrary positive integer m, we have found an infinite set of solutions.

4. Suppose 100 points in the plane are colored using two

colors, red and white, such that each red point is the centre of a circle passing through at least three white points. What is the least possible number of white points?

Solution Let n be the number of white points. Then we can draw at most $\binom{n}{3}$ circles. Thus the number of white and red points together is at most $n+\binom{n}{3}$. We observe that

$$9+\binom{9}{3}=93, \qquad 10+\binom{10}{3}=130$$

Thus $n \geq 10$. We show that $n=10$ works.
Take any 10 points such that no three are collinear and no four are concyclic. Then we get $\binom{10}{3}=120$ points as centre of distinct circles. Among these, there may be some points from the original 10 points. Even if we leave out these 10 points, we have at least 110 points which are centres of circles formed by the ten points we have chosen. Choose any 90 points from them and color them red and color the original 10 points white. We get 100 points of which 10 are white and remaining 90 are red. Each of these 90 red points is the centre of a circle passing through some three white points.

5. In a cyclic quadrilateral $ABCD$ with circumcentre O, the diagonals AC and BD intersect at X. Let the circumcircles of triangles AXD and BXC intersect at Y. Let the circumcircles of triangles AXB and CXD intersect at Z. If O lies inside $ABCD$ and if the points O, X, Y, Z are all distinct, prove that O, X, Y, Z lie on a circle.

Solution Observe that

$$\begin{aligned}\angle AYB &= 360^\circ - \angle AYX - \angle XYB \\ &= (180^\circ - \angle AYX) + (180^\circ - \angle XYB) \\ &= \angle XDA + \angle XCB \\ &= 2\angle ACB = \angle AOB\end{aligned}$$

It follows that A, B, O, Y are concyclic. Similarly,

$$\begin{aligned}\angle CZB &= 360^\circ - \angle CZX - \angle XZB \\ &= (180^\circ - \angle CZX) + (180^\circ - \angle XZB) \\ &= \angle CDX + \angle XAB \\ &= \angle COB\end{aligned}$$

so that C, Z, O, B are concyclic. We have

$$\begin{aligned}\angle XYO &= 360^\circ - \angle XYA - \angle AYO \\ &= (180^\circ - \angle XYA) + (180^\circ - \angle AYO) \\ &= \frac{1}{2}\angle AOB + \left(90^\circ - \frac{1}{2}\angle AOB\right) \\ &= 90^\circ\end{aligned}$$

Similarly, we can show that $\angle XZO = 90^\circ$. It follows that X, Y, O, Z are concyclic.

6. Define a sequence $\{a_n\}$ of real numbers by

$$a_1 = 2, \quad a_{n+1} = \frac{a_n^2 + 1}{2}, \quad \text{for } n \geq 1$$

Prove that $\sum_{j=1}^{N} \frac{1}{a_j + 1} < 1$ for every natural number N.

Solution If $N = 1$, the sum is just $\frac{1}{a_1 + 1} = \frac{1}{3} < 1$.

Hence we may assume that $N > 1$. It is easy to see that $a_j > 1$ for all j. Now,

$$a_{j+1} - a_j = \frac{a_j^2 + 1}{2} - a_j = \frac{(a_j - 1)^2}{2}$$
$$a_{j+1} - 1 = \frac{a_j^2 + 1}{2} - 1 = \frac{(a_j - 1)(a_j + 1)}{2}$$

and hence

$$\frac{1}{a_j + 1} = \frac{1}{2}\left(\frac{a_j - 1}{a_{j+1} - 1}\right)$$

for all $j \geq 1$. Thus

$$\begin{aligned}
\sum_{j=1}^{N} \frac{1}{a_j + 1} &= \frac{1}{2}\sum_{j=1}^{N}\left(\frac{a_j - 1}{a_{j+1} - 1}\right) \\
&= \frac{1}{2}\sum_{j=1}^{N}\left(\frac{(a_j - 1)^2}{(a_j - 1)(a_{j+1} - 1)}\right) \\
&= \sum_{j=1}^{N} \frac{a_{j+1} - a_j}{(a_j - 1)(a_{j+1} - 1)} \\
&= \sum_{j=1}^{N}\left(\frac{1}{a_j - 1} - \frac{1}{a_{j+1} - 1}\right) \\
&= \frac{1}{a_1 - 1} - \frac{1}{a_{N+1} - 1} \\
&< \frac{1}{a_1 - 1} = 1
\end{aligned}$$

INMO 2019

1. Let ABC be a triangle with $\angle BAC > 90^\circ$. Let D be a point on the segment BC and E be a point on the line AD such that AB is tangent to the circumcircle of triangle ACD at A and BE is perpendicular to AD. Given that $CA = CD$ and $AE = CE$, determine $\angle BCA$ in degrees.

   Solution Let $\angle C = 2\alpha$. Then $\angle CAD = \angle CDA = 90^\circ - \alpha$. Moreover, $\angle BAD = 2\alpha$ as BA is tangent to the circum circle of ΔCAD. Since $AE = AD$, it gives $\angle AEC = 2\alpha$. Thus ΔAEC is similar to ΔACD. Hence $\dfrac{AE}{AC} = \dfrac{AC}{AD}$.

   But the condition that $BE \perp AD$ gives $AE = AB\cos 2\alpha = c\cos 2\alpha$. It is easy to see that $\angle B = 90^\circ - 3\alpha$. Using the sine rule in triangle ADC, we get $\dfrac{AD}{\sin 2\alpha} = \dfrac{AC}{\sin(90^\circ - \alpha)}$. This gives $AD = 2bsin\alpha$. Thus we get

$$b^2 = AC^2 = AE \cdot AD = (c\cos 2\alpha) \cdot (2b\sin\alpha)$$

   Using $b = 2R\sin B$ and $c = 2R\sin C$, this leads to

$$\cos 3\alpha = 2\sin 2\alpha \cos 2\alpha \sin\alpha = \sin 4\alpha \sin\alpha$$

   Writing $\cos 3\alpha = \cos(4\alpha - \alpha)$ and expanding, we get $\cos 4\alpha \cos\alpha = 0$. Therefore, $\alpha = 90^\circ$ or $4\alpha = 90^\circ$. But $\alpha = 90^\circ$ is not possible as $\angle C = 2\alpha$. Therefore, $4\alpha = 90^\circ$ which gives $\angle C = 2\alpha = 45^\circ$.

2. Let $A_1B_1C_1D_1E_1$ be a regular pentagon. For $2 \leq n \leq 11$, let $A_nB_nC_nD_nE_n$ be the pentagon whose vertices are the midpoints of the sides of the pentagon $A_{n-1}1B_{n-1}C_{n-1}D_{n-1}E_{n-1}$. All the 5 vertices of each

of the 11 pentagons are arbitrarily colored red or blue. Prove that four points among these 55 points have the same color and form the vertices of a cyclic quadrilateral.

Solution We first observe that all the eleven pentagons are regular. Moreover, there are 5 fixed directions and all the 55 sides are in one of these directions. If we consider any two sides which are parallel, they are the parallel sides of an isosceles trapezium, which is cyclic. If we consider any pentagon, its two adjacent vertices have the same color. Consider all such 11 sides whose end points are of the same color. These are in 5 fixed directions. By pigeon-hole principle, there are 3 sides which are in the same directions and therefore parallel to each other. Among these three sides, two must have end points having one color (again by P-H principle). Thus there are two parallel sides among the 55 and the end points of these have one fixed color. But these two sides are parallel sides of an isosceles trapezium. Hence the four end points are concyclic.

3. Let m, n be distinct positive integers. Prove that

$$\gcd(m,n)+\gcd(m+1,n+1)+\gcd(m+2,n+2) \leq 2|m-n|+1$$

Further, determine when equality holds.

Solution Observe that

$$\gcd(m+j,n+j) = \gcd(m+j,|m-n|)$$

for $j = 0, 1, 2$. Hence we can find positive integers a, b, c

such that

$$\gcd(m,n) = \frac{|m-n|}{a}$$
$$\gcd(m+1,n+1) = \frac{|m-n|}{b}$$
$$\gcd(m+2,n+2) = \frac{|m-n|}{c}$$

It follows that $|m-n|$ divides $ma, (m+1)b, (m+2)c$. Hence we see that $|m-n|$ divides ab and bc. Thus $|m-n| \leq ab$ and $|m-n| \leq bc$. This leads to $b \geq \frac{|m-n|}{a}$ and $b \geq \frac{|m-n|}{c}$. Thus

$$\begin{aligned}\gcd(m,n) + \gcd(m+1,n+1) + \gcd(m+2,n+2) &= \frac{|m-n|}{a} + \frac{|m-n|}{b} + \frac{|m-n|}{c} \\ &\leq 2b + \frac{|m-n|}{b}\end{aligned}$$

We need to prove:

$$2b + \frac{|m-n|}{b} \leq 2|m-n| + 1$$

Taking $|m-n| = K$, we have to show that $2b^2 + K \leq b(2K+1)$. This reduces to $(b-K)(2b-1) \leq 0$. However,

$$K = |m-n| \geq b \geq 1 \geq \frac{1}{2}$$

Equality holds only when $(m,n) = (b, b+1)$ or $(2k, 2k+2)$ or permutations of these for some k.

4. Let n and M be positive integers such that $M > n^{n-1}$. Prove that there are n distinct primes $p_1, p_2, \ldots, p_n$ such that p_j divides $M + j$ for $1 \leq j \leq n$.

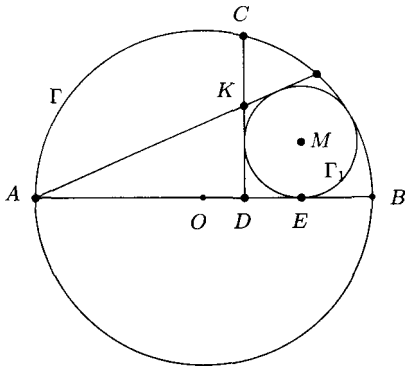
Solution If some number $M+k$, $1 \leq k \leq n$, has at least n distinct prime factors, then we can associate a prime factor of $M+k$ with the number $M+k$ which is not associated with any of the remaining $n-1$ numbers. Suppose $M+j$ has less than n distinct prime factors. Write

$$M+j = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_r^{\alpha_r}$$

where $r < n$. But $M+j > n^{n-1}$. Hence there exists t, $1 \leq t \leq r$ such that $p_t^{\alpha_t} > n$. Associate p_t with this $M+j$. Suppose that p_t is associated with some other $M+l$. Let $p_t^{\beta_t}$ be the largest power of p_t dividing $M+l$. Then $p_t^{\beta_t} > n$. Let $T = \gcd(p_t^{\alpha_t}, p_t^{\beta_t})$. Then $T > n$. Since T divides $M+j$ and $M+l$, it follows that T divides $j-l$. But $|j-l| < n$ and $T > n$, and we get a contradiction. This shows that p_t cannot be associated with any other $M+l$. Thus each $M+j$ is associated with different primes.

5. Let AB be a diameter of a circle Γ and let C be a point on Γ different from A and B. Let D be the foot of perpendicular from C on to AB. Let K be a point of the segment CD such that AC is equal to the semiperimeter of the triangle ADK. Show that the excircle of triangle ADK opposite A is tangent to Γ.

Solution Let R be the radius the circle Γ and r be that of the circle Γ_1, the excircle of triangle ADK. Let O be the centre of Γ and M be that of the circle Γ_1. Let E be the point of contact of Γ_1 with AB. Then $ME = DE = r$. Observe that AE is the semiperimeter of the triangle ADE. We are given that $AC = AE$. Using that $\angle ACB = 90^\circ$, we also get $AC^2 = AD \cdot AB$. Hence $AE^2 = AD \cdot AB$. We have to show that $R - r = OM$ for proving that Γ_1 is tangent



to Γ. We have

$$\begin{aligned} OM^2 - (R-r)^2 &= OE^2 + r^2 - (R-r)^2 \\ &= (AD + DE \cdot AO)^2 + r^2 - (R-r)^2 \\ &= (AD - (R-r))^2 + r^2 - (R-r)^2 \\ &= AD^2 - 2AD \cdot (R-r) + r^2 \\ &= (AD2 + 2AD \cdot r + r^2) - 2AD \cdot R \\ &= (AD + r)^2 - AD \cdot AB \\ &= (AD + DE)^2 - AD \cdot AB \\ &= AE^2 - AD \cdot AB \\ &= 0 \end{aligned}$$

6. Let f be function defined from the set $\{(x, y) : x, y \text{ reals}, xy \neq 0\}$ in to the set of all positive real numbers such that

   (i) $f(xy, z) = f(x, z)f(y, z)$ for all $x, y, xy \neq 0$

   (ii) $f(x, yz) = f(x, y)f(x, z)$ for all $x, y, xy \neq 0$

   (iii) $f(x, 1 - x) = 1$ for all $x \neq 0, 1$

Show that

(a) $f(x,x) = f(x,-x) = 1$ for all $x \neq 0$

(b) $f(x,y)f(y,x) = 1$ for all $x, y \neq 0$

Solution (The condition (ii) was inadvertently left out in the paper. We give the solution with condition (ii).)

Taking $x = y = 1$ in (ii), we get $f(1,z) = f(1,z)^2$ and hence $f(1,z) = 1$, for all $z \neq 0$. Similarly, $x = y = -1$ gives $f(-1,z) = 1$ for all $z \neq 0$. Observe

$$f\left(\frac{1}{x}, y\right) f(x,y) = f(1,y) = 1$$

$$= f(x,1) = f\left(x, \frac{1}{y}\right) f(x,y)$$

Hence

$$f\left(\frac{1}{x}, y\right) = f\left(x, \frac{1}{y}\right) = \frac{1}{f(x,y)}$$

for all $x, y \neq 0$. Now for $x \neq 0, 1$, (iii) gives

$$1 = f\left(\frac{1}{x}, 1 - \frac{1}{x}\right) = f\left(x, \frac{1}{1 - \frac{1}{x}}\right)$$

Multiplying by $1 = f(x, 1-x)$, we get

$$1 = f(x,1-x) f\left(x, \frac{1}{1 - \frac{1}{x}}\right)$$

$$= f\left(x, \frac{1-x}{1 - \frac{1}{x}}\right)$$

$$= f(x,-x)$$

for all $x \neq 0, 1$. But $f(x, -1) = 1$ for all $x \neq 0$ gives

$$f(x, x) = f(x, -x)f(x, -1) = f(x, -x) = 1$$

for all $x \neq 0, 1$. Observe $f(1, 1) = f(1, -1) = 1$. Hence

$$f(x, x) = f(x, -x) = 1$$

for all $x \neq 0$, which proves (a).

We have

$$\begin{aligned} 1 &= f(xy, xy) = f(x, xy)f(y, xy) \\ &= f(x, x)f(x, y)f(y, x)f(y, y) \\ &= f(x, y)f(y, x) \end{aligned}$$

for all $x, y \neq 0$, which proves (b).

ASSOCIATION ACTIVITIES
MINUTES OF THE GENERAL BODY MEETING

Held on 27.12.2018 at the venue of 53rd conference of the AMTI 33 members were present.

1. The meeting commenced with the prayer by Ms. Lakshmi. The President Prof. I.K Rana welcomed the gathering after the General Secretary requested him to preside over the meeting.

2. Minutes of the last General Body meeting was then read by the General Secretary with compliance report or inability to take some steps interspersed. It was adopted unanimously.

3. Audited A/C for the year 2017-2018 was perused by the members and it was also adopted unanimously after few doubts raised and answered by the General Secretary on behalf of the Treasurer. The suggestion to keep fixed fund for Publications and scholarship was agreed to be processed in consultation with the auditor.

4. The following details of Workshop was presented by the General Secretary:

DATE	TITLE	DESCRIPTION
May 14th & 15th 2018	Workshop For Primary Teachers on Introduction to topics through various activities	No of participants-15 Name Of Trainers - Mr. R. Athmaraman, Mr. V. Sundaramurthy.
May 14th & 15th 2018	For Middle and High School Teachers on Use of Geogebra	No of participants-10 Name of Trainer - Dr. S. Muralidharan.
June 7th & 8th 2018	For Students of Grade 7 [Selected students], on Introduction to solving Olympiad papers	No of participants-20, Name Of Trainer - Mr. R. Athmaraman.

DATE	TITLE	DESCRIPTION
June 9th 2018	Workshop For Parents, [selected students], on How to carry forward the Olympiad training for their wards	No of participants- 20, Name Of Trainer – Dr. S. Muralidharan
June 23rd 2018	For Math teachers of grade 9 to 12 , from different schools in Bangalore	No of participants- 30, Name of Trainer – Dr. S. Muralidharan and Mr. M. Mahadevan.

5. President suggested a common email ID for EC members.

6. Mr. R. Athmaraman suggested that we can make the back volumes of our journals available as downloads in our website. Prof. Rana suggested that 2 years back volumes (up to 2016) can be made available.

7. The General Sec. proposed that the additional cost towards acquiring additional space for the website can be worked out by the EC and it was seconded by Mr. Rajesh.

8. There was a request for publication of papers by delegates in the conference as Conference proceedings which was gently rejected by the President, with a remark that we have not reached the required standard to publish conference proceedings.

9. Mr. Tushar Parekh recommended to have volunteers from various states who can popularize mathematics.

10. President responded that a committee to popularize the activities of AMTI may be constituted and the members can present their ideas in the form of proposals to this committee for discussion and decision making.

11. In connection with having Reserved funds for publications, it was decided to use the Reserve funds for journals, which has already been discussed in Executive Committee to take

necessary steps.

12. The President suggested that the Audited accounts may be scanned and shared with all the life members as a soft copy. He also suggested sharing of minutes of Executive Committee meetings with him along with other life members.

13. Dr. R. Santhi expressed her request to be relieved from the post of General Secretary due to her husband's ill health.

14. Dr. M. Palanivasan, the Associate General Secretary, agreed to function as the General Secretary. It was suggested by the President that we can have an Administrative Secretary and an Academic Secretary and Mr. R. Athmaraman suggested that the division of duties can be clearly designed and presented as a formal document. Mr. Athmaraman also added that he has approached our EC Member Dr.Sivaraman who has categorically agreed to be the Academic Secretary

15. It was stated by the President that Mr. Mahadevan can be a permanent Invited member in the EC in the capacity of an Advisor, with the Advisory committee being constituted. He also added that Dr.R. Santhi can be a member in the Advisory committee.

16. In connection with conduct of NMTC and having a district coordinator, it was decided that EC can discuss and come to a consensus.

17. It was decided to send a formal letter to Mr. Mahadevan thanking him for his contribution to AMTI for a long tenure of two decades and inviting him to be an Advisor.

18. The question papers for NMTC may be sent as pdf which is password protected.

19. Google forms can be used for ID letter.

The AMTI Executive Committee meeting held on 30-03-2019, 4 pm, at Sri Sarada Sec. School, Gopalapuram, Chennai 600086.

1. The Executive Chairman Prof. J. Pandurangan, welcomed the members after the General Secretary requested him to

preside over the meeting. He directed the General Secretary to go ahead with the agenda.

2. Conference and related matters: The General Secretary thanked the Conference Secretary Dr. S.R. Santhanam and placed on Record the great contribution from him for organizing a very high quality of Conference at Kota in Dec 2018.

Dr. S. R. Santhanam briefly spoke about the success of Kota conference. He also suggested two venues for our December 2019 conference, namely, CEOA School and NIMSME, Madurai. Sri R. Athmaraman suggested another venue at Tirupathi.

Dr. S.R. Santhanam expressed his grievance that his name was omitted in the Souvenir that was released during 2018 Conference. The General Secretary and Sri R. Athmaraman apologized for the same and it was decided to have a committee for proof reading before printing.

3. May Workshop: The General Secretary provided data related to May workshop registration as follows: Primary -20 SJ-48 RMO group-65 Teachers'-2

4. Status report: a) MT: General Secretary read the email Message received from Dr. S. Muralidharan expressing his inability to attend the EC Meeting as he is out of station, The next issue (Vol. 55, Nos. 1-2) will be ready for printing by 15th April. The General Secretary announced the receipt of a grant of Rs. 75,000/- (Rupees Seventy five thousand) from Department of Science and Technology, Delhi., towards Mathematics teacher.

b) JM: Sri Athmaraman informed that the next issue will be completed within 10 days and he also requested Dr. R. Sivaraman to offer help in the JM related work.

c) NMTC: It was decided to conduct the TC in some states with utmost care to prevent copying. It was further decided to convene an internal meeting to discuss on the above problem.

d) Workshop: The Secretary Project and Workshop Dr. P.

Bakthavatchalu was not present for the Meeting and he has not involved himself either for 2018 or for 2019 May workshops.
It was decided to conduct the May workshop for the teachers only if the teachers' strength is at least 10.
e) Popular Lecture: Mr. P. Ramesh, who is incharge of Popular Lectures said he could not organize any Lecture for this year and he was requested to conduct popular lectures regularly with guidance of Dr. R. Sivaraman.
5. Any other matter:
a) General Secretary also read the mail from Mr. Bhas Bamre stating he could not attend the EC.
b) Our present General Secretary Dr. R. Santhi handed over the charges to the incoming General Secretary Dr. M. Palanivasan, who will function as the General Secretary with effect from 01.04.2019.
c) Dr. R. Sivaraman accepted to take charge as the Academic Secretary and contribute to AMTI in all the possible aspects.
d) Dr. M. Palanivasan suggested that some repair works should be done in our office at Triplicane and Perungalathur.
e) Dr. R. Santhi shared the information that our email ID amti@vsnl.com will be disabled soon as Tata Docomo is closing down.
f) Two minutes silence was observed for the sudden demise of our Executive Member Mr. Bharath Karmarkar from Maharashtra.
6. With tea and vote of thanks by the General Secretary the meeting was concluded.
The AMTI Executive Committee meeting held on 04-04-2019, 4 pm, at Sri Sarada Sec. School, Gopalapuram, Chennai 600086.

NMTC meeting Minutes:
At the outset the problems faced in the conduct and evaluation of NMTC exams were discussed in detail. Suggestions for setting right the existing problems were invited and discussed. The following were the conclusions arrived at:

1. RMO/INMO is currently not under the purview of AMTI as it was earlier. It is completely centralized and conducted by NBHM.
2. Applications for NMTC all four levels must be submitted online only. The schools have to submit the consolidated applications of the students to AMTI online. No postal or telephonic communication will be entertained.
3. The set of languages in which the papers will be translated remains the same. No change.
4. The fee is increased to Rs 75 per applicant from 2019 onwards.
5. As we found a lot of malpractices in the open quota registrations we have dropped open quota from this year. The students can apply online to AMTI if their school does not conduct NMTC, and they will be allotted to the nearest centre. It is the responsibility of the applicant and parents to take the exam in the allotted centre. The centre once allotted will not be changed and no such request will be entertained.
6. Each school can send exactly 10% of the students registered/appeared for the next level. Any ties have to be resolved by the school. The school will be provided the deadline for exactly 10% of registered candidates and they have to fill the details of the students and send it back to AMTI. No email communication in this regard asking for increasing the number of selected candidates due to equal marks will be entertained.
7. In AP we need to look out for common centres. The students must be allotted to the nearest common centre for writing the final exam.
8. The 10% list of students who take the final exam and the answer papers of the final exam will not be accepted if they are received after the deadline.

A meeting has to be conducted for finalizing the common centres.

The AMTI Executive committee meeting held on 18-05-2019

at our project office, Perungalathur.
Our Executive Chairman Prof. J. Pandurangan, presided over the meeting. The General Secretary welcomed the members. The General Secretary took up the points one after the other.
1. Since the audited accounts and budget for 2018-2019 are not finalized by auditor's office the same could not be submitted, in the E.C. for approval. Dr. S.R. Santhanam requested to check thoroughly the expenses for Kota conference.
2. Status reports: a) MT:- MT Editor Dr. S. Muralidharan sent a mail to our office regarding the Association activities article and it will be prepared and send to the editor for approval.
b) JM:- JM Editor Sri R. Athmaraman informed that the next issue will be ready by first week of June 2019.
c) Workshop: Dr. M. Palanivasan and Sri V. Sundaramurthy participated as the resource persons in student workshop held at Bajaj Science Centre, Wardha on 19th and 20th April 2019. Sri R. Athmaraman and Sri S. Rajesh participated as resource persons in the teachers workshop at Jain Heritage School, Bangalore on 14th and 15th May 2019.
d) Regarding May 2019 Summer Workshops students details are Primary -37 Date (01.05.2019 to 04.05.2019) Sub-Junior -75 Date (08.05.2019 to 11.05.2019) Junior (RMO)-107 Date (15.05.2019 to 18.05.2019) All the above three categories workshops went on very well. We could not conduct the teachers workshop this year because of poor registration.
e) NMTC: As per the request given by the Talent examination Secretary Dr. Hemalatha Thiagarajan separate meeting was conducted for NMTC 2019. The Registration fees was fixed as Rs. 75/- out of which Rs. 15 to be retained by the institution and Rs. 60/- to be submitted to AMTI. As we found a lot of problems in the open quota registrations we have dropped open quota from this year. The students can apply online to AMTI if their school does not conduct NMTC and they will be allotted to the nearest centre the centre once allotted will

not be changed and no such request will be entertained. Each school can send exactly 10% of the students appeared for the next level. Any ties have to be resolved by the school. In AP and other places we need to look out for common centre. The students must be allotted to the nearest common centres for writing the final examination. The General Secretary requested Dr.S.R.Santhanam,
Mr.G.Lakshmanamoorthy, Mr.Y.Narasihma Raju, Mr. Bhas Bamre and Mrs. S.Jayapriya to identify the common centres. All the points were discussed in details Dr. S.R. Santhanam expressed his support to conduct the examination without Malpractice and Sri R. Athmaraman stated that coping should be viewed seriously. Dr. Hemalatha Thiagarajan expressed that the malpractices Institutions should be block listed. Dr. M. Kumaraswamy expressed that code A,B,C,D question papers can be used. But the General Secretary stated that practically it would be difficult to have code A,B,C,D question papers Dr. M. Kumaraswamy requested to fix the curriculum for each group. (Primary, Sub-Junior, Junior and Inter).
f) Popular Lectures:- Popular lecture Secretary Mr. P. Ramesh was requested to organize minimum 4 lectures per year.
g) Conference:- i. Sri R. Athmaraman informed about the Time 2019 conference form 7 to 10 December 2019 which will have our president Prof. Inder K. Rana as a convener.
ii. Our conference Secretary Dr. S.R. Santhanam suggested to conduct our AMTI 54th Annual Conference at NIMSE, Hyderabad.
He would inform the details in the second week of June 2019 and the call notice may be given in the last week of June.
3. Any other matter:-
i. Dr. R. Sivaraman suggested to conduct a workshop for Govt. School teachers or Govt. school students.
ii. Dr. S.R. Santhanam suggested to conduct one more ESTEEM workshop which was successfully organized by Sri.

R. Athmaraman some years ago.

iii. Dr. M. Kumaraswamy suggested to fix photos of mathematicians in all the rooms at our project office.

4. Some of the Executive Committee members requested the General Secretary to review the previous year's expenditure records. After the verification of records for the years 2016-17 and 2017-18, the General Secretary found that almost all the orders were given to one particular company M.M. System, Kodambakkam, Chennai. After discussions on the performance of the vendor, the Executive Committee members unanimously decided not to give any order hereafter to M M System from AMTI.

5. The meeting concluded with the vote of thanks given by Sri P. Ramesh.

ASSOCIATION ACTIVITIES

Reports of 53rd Annual Conference, Kota

Day I

The 53rd Conference of our Association was hosted by Disha Delphi Public School, Kota (Rajasthan), on 26th, 27th and 28th December 2018 in a grand manner. There were 170 registrations and 160 attended. Followed by the inaugural function starting with prayer, release of souvenir and addresses by Prof Neelima Singh, Vice Chancellor, University of Kota and Shri V K Jain, Site Director, NPCIL-RR SITE, Rawatbhata.

Prof. I.K. Rana, our respected president, gave his address providing awesome ideas to further the goals of the AMTI. This was followed by Prof. A. Narasinga Rao memorial lecture, a spectacular one, on "Teaching Mathematics through Math Lab Activities with Low Cost / No Cost Teaching Aids" By Prof. Dr. Hukum Singh, Professor of Mathematics , Former Dean Academics and Head Department of Education in Science and Mathematics and Division of Educational Kits in the NCERT, New Delhi. (It may be recalled that this lecture is always devoted to Math Education at School Level).

Paper Presentation sessions, held in three halls, were chaired by Dr. S. R. Santhaman, Prof. J. Pandurangan (our Executive Chairman) and Sri R. Athmaraman.

A small but quite elegant and educative math Exhibition was neatly organized by the hosts, the contents of which were profusely appreciated by participants.

Post lunch session was dominated by another larger session on paper presentations. The final programme was a written test, a prelude to the Oral Quiz scheduled to be held on the final day.

The first day ended with Local sight seeing arranged by the hosting school.

Day II

Day 2 commenced with Presentations by students. The second item in the sequence was Prof. P.L.Bhatnagar Memorial Lecture (usually dealt on applied mathematics) on "AN INTRODUCTION TO MECHANICS OF FLUIDS" by Prof. Anil Sharma. Next, an invited talk on "Heuristics of Problem solving" by Dr. S.R. Santhanam (Conference Secretary) enlightened the participants with essentials of the techniques. The third phase of Paper Presentations followed these.

The subsequent talk by Dilip Gotkhindikar on "History of Mathematics" was a marvel and formed Prof.R.C. Gupta Endowment Lecture (sponsored by our former president).

Panel Discussion on the Theme of the conference was the next item where many interesting observations were made.

Then the General body of the AMTI met with 33 members attending, in which were passed the minutes of previous meeting, audited annual accounts 2017-2018 etc.

Cultural Programme by the Host School was the concluding fascinating event for the day.

Day III

On the third day 16 delegates consisting of 4 teams were selected, based on their written quiz performance and participated in the oral quiz led by quiz master Sri R. Athmaraman. The winners were given Prizes in the Valedictory function with Rs.1000/- each for 4 as 1st prize and Trophy, Rs. 750/- each for 4 as 2nd prize and Trophy, Rs. 500/- each for 4 as 3rd prize from the endowment created by Prof. V. Krishna Murthy, formerly of BITS, Pilani in the name of his illustrious Prof. K. Ganapathy Iyer.

Short communications and student presentations threw light on several important pedagogical problems. After the final session of Student presentations, an open session was held to obtain feedback from participants and also suggestions for further improvement of the proceedings.

The Valedictory Function had Shri V K Jaitly, SR. Vice President & Factory Head, DCM Shriram Rayons, Kota as Chief guest who enlightened the audience on life-oriented mathematics. The Guest of honour for the function, Mr. V K Bansal Chief Managing Director, Bansal Classes Pvt Limited, Kota provided a beautiful speech to complete the function.

With tea break and issue of participation certificates, the grand conference came to an end.

ASSOCIATION MEMBER ACTIVITIES

1. The conference secretary Dr S R Santhanam delivered an invited plenary lecture in the international conference of Basic Education conducted by the Mathematics Teachers Society, Philippines during May 3rd to 5th.

2. Professor Hemalatha Thiagarajan has accepted the invitation from HBCSE to be the Associate Coordinator for Regional Mathematical Olympiads for Tamilnadu and Pondicherry regions.

3. Dr S Muralidharan has accepted the invitation from HBCSE to be the Chief Examination Coordinator for Mathematical Olympiads conducted by NBHM/HBCSE. He continues to be the Regional Coordinator for Tamilnadu and Pondicherry regions.

51st National Mathematics Talent Contests (NMTC)

Last date for entries to reach the AMTI	:	15.07.2019
Question packets/ message on it from the AMTI to reach the institution, latest by	:	10 a.m. 30-08-2019
Preliminary Test	:	**31st August 2019** **(Saturday) 2 p.m. to 4 p.m.**
10% best results to reach the AMTI	:	05-09-2019
Final Test	:	**2nd November 2019** **(Saturday) 1 p.m. to 4 p.m.**
Dispatch of Answer Books to The AMTI	:	Same day (only by Speed post)
Announcement of Results	:	By the end of the December 2019

Dr. T. Hemalatha,
Secretary for Talent Exam.

Dr. M. Palanivasan,
General Secretary

More details see our website : **amtionline.com**

AMTI - 53rd ANNUAL CONFERENCE AT KOTA.

The Association of Mathematics Teachers of India

Office: B 19, Vijay Avenue, 85/37 Venkatrangam Street, Triplicane, Chennai - 600 005. Ph: 2844 1523.
E-mail: support@amtionline.com Website: amtionline.com

PRESIDENT	**Dr. INDER K.RANA** Department of Mathematics I.I.T., Bombay, Powai, Mumbai - 400076. Mobile: ikrana@gmail.com
EXECUTIVE CHAIRMAN	**Prof. J. PANDURANGAN** New No.3.Old No 70-A, Kalaimagal Nagar, Third Main Road, Ekkattuthangal, Chennai-600097. Phone: 044 -22250572
GENERAL SECRETARY	**Dr. M. PALANIVASAN,** Plot No:24,Lakshmi Street, Balaiah Garden, Madipakkam, Chennai-600091 Mobile: 9444307602
TREASURER	**Sri V. SUNDARAMURTHY** 529/5, Lake View Main Road, Ayyappa Nagar Extension, Madippakkam, Chennai Mobile: 9840410545 / 9841070392
EDITOR - Mathematics Teacher	**Dr. S. MURALIDHARAN** 18, Phase 4, Wood Creek County, St. Thomas Mount P.O., Chennai-600016 drsmuralidharan@gmail.com
EDITOR - Junior Mathematician	**Sri R. ATHMARAMAN** 35, Venkatesa Agraharam, Mylapore, Chennai – 600004. Mobile: 9444611066
SECRETARY- Talents Competition	**Dr. HEMALATHA THIAGARAJAN** Block No:T8, Door No:TF35, Ashiana Shubham - Phase , Govindapuram,Melrosapuram Road Maraimalai nagar, Singaperumal Kovil (P.O) Chennai-603204. Mobile:9443837585
SECRETARY- Conference	**Dr. S.R. SANTHANAM,** 30/2, Senthil Andavar Street Dhanalakshmi Colony, Vadapalani Chennai- 600026 Phone:044 - 23620539
SECRETARY ACADEMIC	**Dr. R. SIVARAMAN,** 9/11, South Gangai Amman Koil, 2nd Street, Four Squares, Choolaimedu, Chennai- 600094 Phone: 044-24735619 / 7845014341
SECRETARY- Workshops & Projects	**Dr. P. BAKTHAVATCHALU,** 374, Avvai Street, Belliyappa Nagar, SBI Colony Walajapet, Vellore Dist - 632513 Mobile: 9380194801
SECRETARY- Popular Lecture	**Sri P. RAMESH** Plot No:134,First Street Kamaraj Avenue Sithalapakkam, Chennai - 600126 Mobile: 9940060644

THE ASSOCIATION OF MATHEMATICS TEACHERS OF INDIA

(Founded 1965; Registered as S.No.143 of 1965 under the Societies Registration Act XXI of 1860)

Price List of Available Publications

1. Sample Question and Solution for Sub Junior Level	Rs. 80/-
2. Gems Junior	Rs. 125/-
3. Gems Inter	Rs. 125/-
4. The charm of Problem Solving Revised and Enlarged Edition	Rs. 100/-
5. Gems from the Mathematics Teacher	Rs. 60/-
6. Non Routine Problems in Mathematics(with Solutions)	Rs. 100/-
7. Triangles	Rs. 30/-
8. A Treatise on Problems of Maxima and Minima Solved by Algebra	Rs. 100/-
9. Creativity of Ramanujan Primary	Rs. 50/-
10. Creativity of Ramanujan Middle	R
11. Mathematic Muse	F
12. Development of Mathematical Thinking	F
13. The wonder world of Kaprekar Numbers	
14. Mathematics and Magic Squares	
15. Innovative Strategies of Teaching School Mathematics (Primary)	
16. Two Mathematical Offerings	
17. Gems Primary II	
18. Gems Sub Junior II	
19. Gems Junior II	
20. Gems Inter II	
21. RMO-INMO Olympiad Problems with Solutions II	
22. Mathematical Expositions	

****

www.amtionline.com